PAPER

The Modified Multistage Decoding Scheme (MMDS) for a Fast Frequency-Hopped Multiple Access MFSK System over a Rayleigh Fading Channel

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SUMMARY The stage 3/2 decoding scheme, originally suggested by U. Timor [3], is modified for a Rayleigh fading channel to improve the performance of a fast frequency-hopped multiple access/multilevel frequency shift keying system. When signal-to-noise ratio per bit is 30 dB, the simulation results show that the modified stage 3/2 decoding scheme increases the spectral efficiency by 11% compared to the modified stage 1 decoding scheme at bit error rate of 10^{-3}. Further, the performance comparisons are made between the modified multistage decoding scheme and the diversity combining methods, where the modified stage 3/2 decoding scheme shows better performance.

key words: *FFHMA/MFSK, MMDS (modified multistage decoding scheme), DCM (diversity combining methods)*

1. Introduction

There have been many investigations to improve the performance of a fast frequency-hopped multiple access/multilevel frequency shift keying (FFHMA/MFSK) system. The inherent diversity of the FFH systems can provide some protection against multiple access (MA) interference, noise and fading. To make a symbol decision with diversity at the receiver, we need a processing, termed the diversity combining. Many diversity combining methods (DCMs) have been proposed for the FFHMA/MFSK systems [1].

An alternative approach to resolve the MA interference problem for the FFHMA/MFSK system is the multiuser detection algorithm, which can eliminate the interference by using the information of other users, thus it is more complex than DCMs. By taking advantages of the well-defined algebraic structure of the address pattern construction, a multiuser detection algorithm referred to the stage 2 decoding was proposed for the FFHMA/MFSK system, which is capable of eliminating most of the interference [2]. An intermediate stage of decoding (stage 3/2), which uses other users' decoded messages, was presented to further improve the performance of the stage 2 decoding by distinguishing the actual interference from the pseudo interference [3]. Noted that those decoding schemes were considered in a noise-free channel, the stage 2 decoding scheme was modified to include the effects of noise and fading [4].

In this paper, the stage 3/2 decoding scheme is modified for a Rayleigh fading channel. Then the performance of a FFHMA/MFSK system with the modified multistage decoding scheme (MMDS), which includes the modified stage 2 decoding scheme and the modified stage 3/2 decoding scheme, is evaluated over a Rayleigh fading channel, and compared with that of the DCMs. In the following, the system model is described in Sect. 2. The procedures of the MMDS are explained in Sect. 3, followed by the simulation results in Sect. 4. Finally, the conclusions are made in Sect. 5.

2. System Model

The block diagram of a FFHMA/MFSK system suggested by Goodman, et al. [5] is shown in Fig. 1. The data stream of rate R_b bits/second is mapped into an M-ary FSK symbol of rate R_s symbols/second, where $R_s = 1/T_s = R_b/k$ and $k = \log_2 M$. One M-ary symbol is transmitted on L hops. Thus the chip duration is $\tau = T_s/L$ seconds and the chip rate is $R_c = 1/\tau$. The bandwidth W needed to support $M = 2^k$ tones, that are orthogonal over τ seconds, is $W = M/\tau = 2^k L/T_s$ Hz. Therefore, the length of diversity, L, is determined by $L = rk2^{-k}$, where $r = W/R_b$. Perfect time and frequency synchronization is assumed, and $P_b \leq 10^{-3}$ is

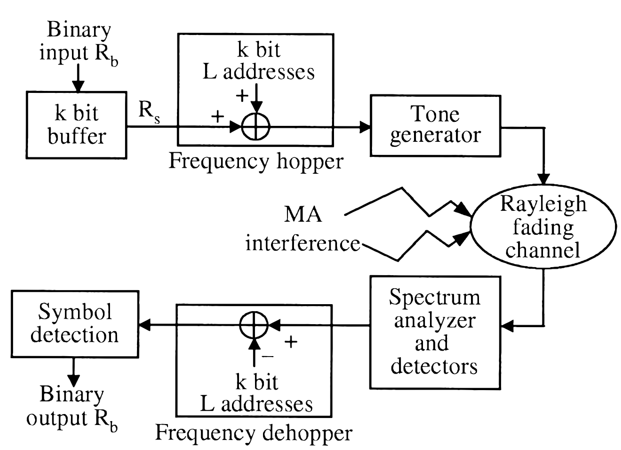

Fig. 1 Block diagram of a FFHMA/MFSK system.

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chosen as the bit error rate (BER) performance criterion because it is sufficient to guarantee the adequate voice transmission quality. Let the system capacity K be the number of concurrent users that can be accommodated at a given bit error probability, $P_b = 10^{-3}$, then the spectral efficiency η of the system is defined as $\eta = KR_b/W = (K/r)$ bits/s/Hz.

Each user is given a distinct address (hopping pattern). The mth user is assigned an address $\vec{a}_m = \left(\gamma_m, \gamma_m\beta, \gamma_m\beta^2, \cdots, \gamma_m\beta^{L-1}\right)$ which consists of L components of $GF(2^k)$ where $GF(2^k)$ is a Galois field over 2^k, and γ_m is the element of $GF(2^k)$ assigned uniquely to the mth user, and β is a primitive element of $GF(2^k)$ which is fixed for the system [6]. The transmitted sequence $\vec{Y}_m = (y_{m0}, y_{m1}, \cdots, y_{mL-1})$ is determined by modulating the k-bit message $\vec{X}_m$ onto the user's address $\vec{a}_m$, it is achieved by addition of the address and the message as $\vec{Y}_m = \vec{a}_m + \vec{X}_m$, where the operations of addition and multiplication of all elements are performed according to the rules of $GF(2^k)$.

Considering the effects of additive white Gaussian noise (AWGN) and Rayleigh fading, where the Rayleigh fading channel is assumed to be flat during a chip duration and each chip is assumed to fade independently, the received waveform is modeled as

$$\hat{r}(t) = \hat{d}_m(t) + \sum_{z=1,\, z\neq m}^{K} \hat{I}_z(t) + n(t) \tag{1}$$

where $\hat{d}_m(t)$ is the desired signal, $\hat{I}_z(t)$ is the interference caused by the zth interferer, and $n(t)$ represents AWGN with double-sided power spectral density $N_o/2$. The desired signal can be written as

$$\begin{aligned}\hat{d}_m(t) \;=& \sum_{l=1}^{L} \alpha_l \sqrt{\frac{2E_c}{\tau}} P_\tau(t - l\tau) \\ &\cdot \cos\Big[2\pi\Big(f_0 + \frac{y_{ml}}{\tau}\Big)t + \theta_l\Big]\end{aligned} \tag{2}$$

where $\{\alpha_l\}$s are independent and identically distributed *(i.i.d.)* Rayleigh random variables with $E[\alpha_l^2] = 1$, $\{\theta_l\}$s are *i.i.d.* random variables that are uniformly distributed within the interval $[0, 2\pi)$, E_c is the average signal energy per chip, f_0 is the lowest frequency, and

$$P_\tau(t) = \begin{cases} 1, & 0 \le t < \tau\,; \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The signal of the zth interferer is modeled as

$$\begin{aligned}\hat{I}_z(t) \;=& \sum_{n=1}^{M}\sum_{l=1}^{L} \alpha_{znl} \sqrt{\frac{2E_c}{\tau}} P_\tau(t - l\tau) \\ &\cdot \cos\Big[2\pi\Big(f_0 + \frac{n}{\tau}\Big)t + \theta_{znl}\Big]\end{aligned} \tag{4}$$

where $\{\alpha_{znl}\}$ and $\{\theta_{znl}\}$ are amplitude and phase, respectively, of the zth interference in the lth column and the nth row of the time-frequency matrix, which are modeled to be *i.i.d.* random variables and have the same distributions as $\{\alpha_l\}$ and $\{\theta_l\}$, respectively. Note that there is only one non-zero α_{znl} (or θ_{znl}) for each (z, l) pair.

After frequency dehopping with the address that is used at the frequency hopper, the dehopped signals are passed through the M parallel energy detectors.

3. Modified Multistage Decoding Scheme (MMDS)

The modified stage 2 decoding scheme was proposed for a FFHMA/MFSK system to reduce the effects of the MA interference over a Rayleigh fading channel [4]. The stage 3/2 decoding scheme that improved the performance of the stage 2 decoding scheme in a noise-free channel is modified for a Rayleigh fading channel. The multiuser detection algorithm, named as the MMDS, includes the modified stage 2 decoding scheme and the proposed modified stage 3/2 decoding scheme.

The flow chart of the MMDS is described in Fig. 2. To detect the symbol with the MMDS, each energy detector is followed by a threshold device with a fixed threshold to make a hard decision. A hard decision is made on each output value of the energy detector, z_{mi}, based on a fixed threshold, E_t, as

$$\begin{aligned}&\hat{z}_{mi} = \begin{cases} 1, & z_{mi} \ge E_t \\ 0, & z_{mi} < E_t \end{cases}; \\ &m = 1, 2, \cdots, M,\; i = 1, 2, \cdots, L\end{aligned} \tag{5}$$

where E_t is the optimum threshold for a decision of an on-off keying symbol in Rayleigh fading [7]. Then $\hat{z}_{mi}$ are used in each decoding procedure.

3.1 Modified Stage 1 Decoding Procedure

In a Rayleigh fading channel, the received matrix of the mth user is corrupted by insertions and deletions of entries, and the correct row, X_m, might be incomplete in the decoded matrix of the mth user, A_m. Therefore, the row with the maximum number of entries should be decoded as the correct message.

After L chips, the $\hat{z}_{mi}$ in each of the M rows are summed to obtain the decision metrics

$$q_m = \sum_{i=1}^{L} \hat{z}_{mi}\,; \quad m = 1, 2, \cdots, M. \tag{6}$$

Then the row which has the largest decision metric, q_m, ('largest' row) is decoded as the correct message. The counted number of entries of the 'largest' row is defined as CN, that is equal to the largest decision metric, q_m. If there are R, $(R \ge 2)$, 'largest' rows, the modified stage 2 decoding procedure is required.

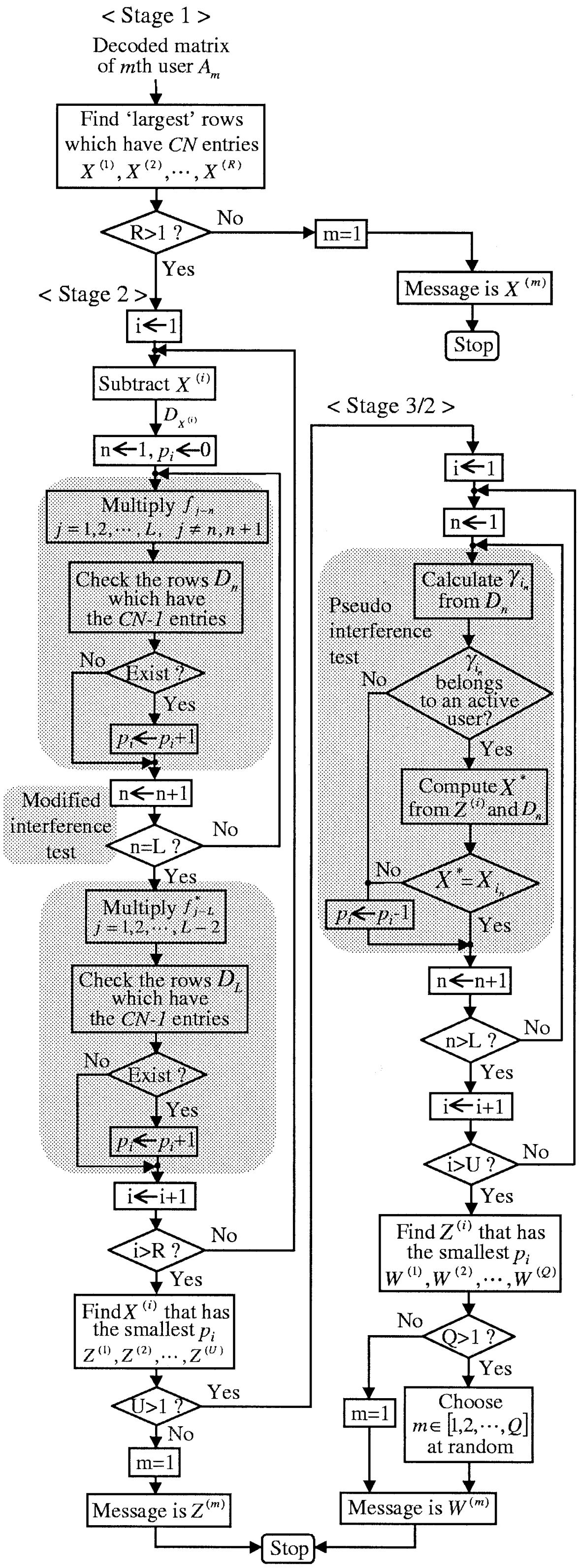

Fig. 2 Flow chart of the MMDS.

3.2 Modified Stage 2 Decoding Procedure

The stage 2 decoding was suggested to provide a systematic procedure that tests whether a complete row is the correct row or the interference row [2]. The stage 2 decoding requires a small increase in complexity (of the order of L^2) while achieving a substantial improvement in the performance. However, it is probable that there is no complete row in the decoded matrix with the effects of noise and fading, thus the stage 2 decoding was modified as shown in the left column of the flow chart in Fig. 2 [4].

If there are R, $(R \geq 2)$, 'largest' rows, except one 'largest' row, X_m, that comes from the desired user, another $(R-1)$ 'largest' rows are the interference rows. The modified stage 2 decoding can distinguish and eliminate those interference rows. According to the address assignment, each chip in an interference row must come from a different interferer, thus at least CN interferers have combined to make this interference row. The interferer who contributes the interference row in the nth column is denoted by i_n. First of all, we subtract the number of the 'largest' row $X^{(i)}$ from all rows in A_m to obtain a new matrix $D_{X^{(i)}}$, $(i = 1, 2, \cdots, R)$, for each 'largest' row. Let $d_n(j)$ be the entry that appears in $D_{X^{(i)}}$ at row $d_n(j)$ and column j, $(j = 1, 2, \cdots, L)$. Then, we can derive the following relations [2]:

$$\begin{aligned} D_n &\equiv d_n(n+1) = f_{j-n} d_n(j),\ n = 1, 2, \cdots, L-1, \\ D_L &\equiv d_L(L-1) = f^*_{j-L} d_L(j), \end{aligned} \tag{7}$$

where

$$\begin{aligned} f_{j-n} &\equiv \frac{\beta - 1}{\beta^{j-n} - 1},\quad j = 1, 2, \cdots, L,\quad j \neq n, n+1, \\ f^*_{j-L} &= \frac{1}{\beta} f_{j-L}, \qquad j = 1, 2, \cdots, L-2. \end{aligned} \tag{8}$$

In order to perform the modified interference test, we multiply all entries in column j of $D_{X^{(i)}}$ by f_{j-n}, $(n = 1, 2, \cdots, L-1)$, or f^*_{j-L}, $(n = L)$, and then look for the rows D_n, $(n = 1, 2, \cdots, L-1)$, or D_L, $(n = L)$. If there exists such a row, the nth chip passes the modified interference test.

The modified interference test is performed on each chip of each row among R 'largest' rows. Let p_i, $(i = 1, 2, \cdots, R)$, be the number of the chips that pass the modified interference test for each 'largest' row. Finally, the 'largest' row that has the smallest p_i is decoded as the correct message. If U, $(U \geq 2)$, 'largest' rows have the same smallest p_i, then the modified stage 3/2 decoding procedure is needed.

3.3 Modified Stage 3/2 Decoding Procedure

In the stage 2 decoding procedure, if all complete rows pass the interference test, the stage 2 decoding fails to identify the correct message, X_m. There are two possible situations: (1) The correct row coincides with an interference row. Hence there is no way to distinguish the correct row, X_m, from any other interference rows. (2) Only $L-j$ chips of X_m are the result of the interference, and the remaining j chips pass the interference test although they are not caused by the interference. This kind of the interference is called a pseudo interference, which is made by the combination of entries from the different interferers at those remaining j chips. If those pseudo interferences can be identified for at least one chip of X_m, X_m will be decoded as the correct message. The pseudo interference test will be done by making use of the available information of other users, derived but not used in the stage 2 decoding procedure. Since the messages of the possible interferers are decoded up to the stage 2 decoding while the message of the desired user, X_m, is decoded up to the stage 3 decoding, it is defined as the stage 3/2 decoding procedure. In the base station, all active users' messages are already decoded up to the stage 2 decoding, subsequently, the modified stage 3/2 decoding procedure does not require extra complexity. However, in the mobile station, it is noteworthy that the complexity increases exponentially as more and more users have to be decoded. Considering noise and fading, this stage 3/2 decoding is modified as shown in the right column of the flow chart in Fig. 2.

If there are U, $(U \geq 2)$, 'largest' rows, $Z^{(i)}$, $(i = 1, 2, \cdots, U)$, which have the same smallest p_i, the pseudo interference test is performed on each chip of each row among U 'largest' rows. First, we calculate γ_{i_n} from D_n, which is found during the modified interference test, and the following equations:

$$D_n = \begin{cases} (\gamma_{i_n} - \gamma_m)\left(\beta^n - \beta^{n-1}\right), & n = 1, \cdots, L-1, \\ (\gamma_{i_L} - \gamma_m)(\beta^{L-2} - \beta^{L-1}), & n = L. \end{cases} \tag{9}$$

Assume that the list of the addresses of the active users is known to the base station. If the address of a possible interferer, γ_{i_n}, is not in the list of the active users, that interference is immediately identified as a pseudo interference and then eliminated from the interference chips, p_i. Otherwise, the computed message value of that interferer, X^*, is compared with the transmitted message value of that interferer, X_{i_n}, which is decoded up to the modified stage 2 decoding, where X^* is obtained by [3]

$$X^* = \begin{cases} Z^{(i)} - \frac{D_n}{\beta - 1}, & n = 1, \cdots, L-1, \\ Z^{(i)} - \frac{D_L \beta}{1 - \beta}, & n = L. \end{cases} \tag{10}$$

If the result is the same, the interference will be identified as an actual interference, otherwise, a pseudo interference and then eliminated from the interference chips, p_i. In this way, the row that has the smallest p_i is decoded as the correct message. If there are Q, $(Q \geq 2)$, 'largest' rows that have the same smallest p_i, the symbol is selected at random from those tied rows.

4. Simulation Results

The bit error rate (BER) performance of a FFHMA/MFSK system that uses the MMDS is simulated over a Rayleigh fading channel. The total bandwidth is $W = 20$ MHz and the bit rate per user is fixed as $R_b = 32$ kbps. For a given W, R_b, and P_b, an optimum number of bits that maximizes the spectral efficiency is obtained as $k = 9$. Then the number of hops per symbol is obtained as $L = 11$ according to $L = rk2^{-k}$ where $r = W/R_b = 625$. When we use k smaller than 9 for the same r, there is slight performance degradation under the same fading condition, similar like in [4]. The simulation results of the BER performance over a Rayleigh fading channel with $E[\alpha^2] = 1$ are shown in Fig. 3 when signal-to-noise ratio per bit (E_b/N_o) is 20 dB and 30 dB. The modified stage 3/2 decoding scheme improves the BER performance compared to the modified stage 2 decoding scheme. In addition, as E_b/N_o increases, the performance improvement by the stage 3/2 decoding scheme also increases. Table 1 lists the simulation results of two important system parameters, the system capacity (K) and the spectral efficiency (η) at $P_b = 10^{-3}$. The spectral efficiency of the modified stage 3/2 decoding scheme at $E_b/N_o = 20$ dB is almost the same as that of the modified stage 1 decoding scheme at $E_b/N_o = 25$ dB. The spectral efficiency of the modified stage 3/2 decoding scheme at $E_b/N_o = 25$ dB is 34%, while that of the modified stage 1 decoding scheme at $E_b/N_o = 30$ dB is 30%. From these results, it can be concluded that E_b/N_o can be saved about 5 dB by using the modified stage 3/2 decoding scheme without requiring a significant increase in complexity compared to the modified stage 1 decoding scheme.

Next, we investigated the influence of the Rayleigh

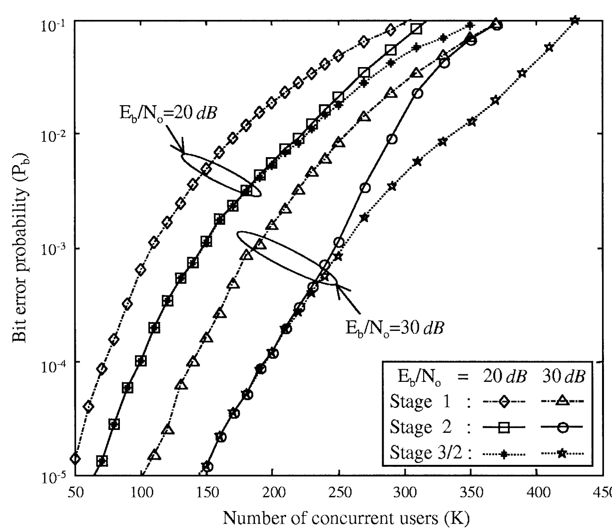

Fig. 3 BER performance vs. number of users for $E[\alpha^2] = 1$, $E_b/N_o = 20$ dB and 30 dB, $k = 9$, $L = 11$ and $r = 625$.

Table 1 Capacity (K) and spectral efficiency (η) with $W = 20\,\text{MHz}$, $R_b = 32\,\text{kbps}$, $k = 9$, $L = 11$ and $P_b = 10^{-3}$.

Rayleigh fading channel	Decoding scheme	Capacity (K)	Spectral efficiency (η)
$E_b/N_o = 20\,\text{dB}$	Stage 1	107	17%
	Stage 2	148	24%
	Stage 3/2	148	24%
$E_b/N_o = 25\,\text{dB}$	Stage 1	156	25%
	Stage 2	208	33%
	Stage 3/2	210	34%
$E_b/N_o = 30\,\text{dB}$	Stage 1	187	30%
	Stage 2	246	39%
	Stage 3/2	255	41%

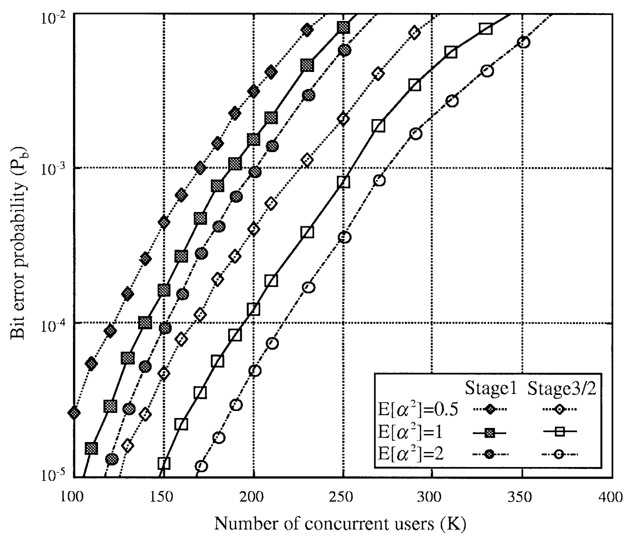

Fig. 4 BER performance vs. number of users for various fading conditions, $E_b/N_o = 30\,\text{dB}$, $k = 9$, $L = 11$ and $r = 625$.

fading parameter on the BER performance of the modified stage 3/2 decoding scheme. Figure 4 depicts the simulation results of the BER performance with $E[\alpha^2] = 0.5, 1$ and 2, when E_b/N_o is 30 dB. As $E[\alpha^2]$ increases, the effects of fading becomes weaker. From these curves, we found that the modified stage 3/2 decoding scheme improves the performance monotonically as the fading factor decreases. In addition, the performance improvement by the modified stage 3/2 decoding scheme compared to the modified stage 1 decoding scheme is larger in the negligible fading condition than in the severe fading condition. Figure 5 shows that the case where users have unequal power distributions due to the near-far effect. The 'worse case' is that the interferers are divided into three groups whose powers are ten times, five times and twice as large as the desired signal, respectively. For each group of the 'better case', the signal-to-interference power ratios are the reciprocals of those for the worse case, respectively. We observe that the performance of the modified stage 3/2 decoding scheme is quite insensitive to the power variations of the interferers.

Subsequently, to analyze the advantage of the complex MMDS over the simple DCMs, the BER performance of the MMDS is compared with that of the DCMs over a Rayleigh fading channel. Three diversity combining techniques, normalized envelope detection (NED), reduced rank sum (RRS), order statistics-normalized envelope detection (OS-NED) are examined. This time, the values of k and L are determined as 4 and 8, respectively where $r = W/R_b = 32$ [1]. Figure 6 and Fig. 7 show the simulation results of the BER performance when signal-to-noise ratio per chip ($E_c/N_o = (E_b/N_o)(k/L)$) is 25 dB and 35 dB, respectively. The OS-NED DCM and the MMDS show better performance than other methods. The modified stage 3/2 decoding scheme has the best performance, and

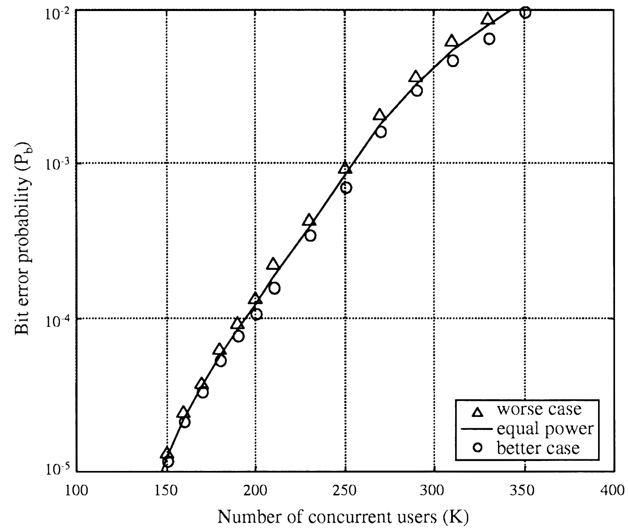

Fig. 5 BER performance of the modified stage 3/2 decoding scheme vs. number of users with the effect of unequal power condition, $k = 9$, $L = 11$ and $r = 625$.

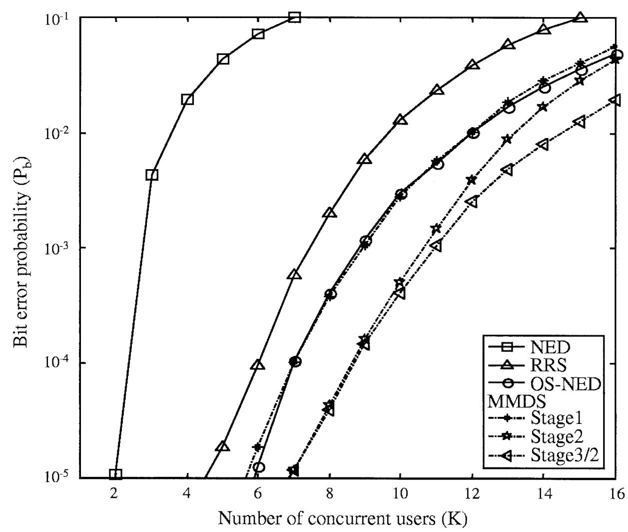

Fig. 6 BER performance vs. number of users for three diversity combining schemes and MMDS, $E_c/N_o = 25\,\text{dB}$, $k = 4$, $L = 8$ and $r = 32$.

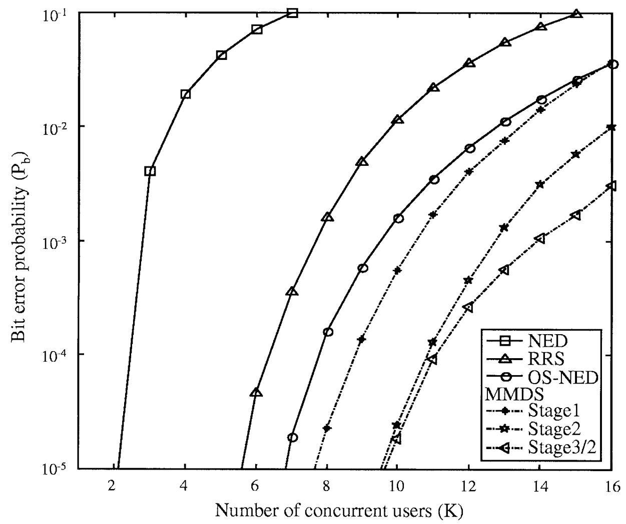

Fig. 7 BER performance vs. number of users for three diversity combining schemes and MMDS, $E_c/N_o = 35$ dB, $k = 4$, $L = 8$ and $r = 32$.

improves the BER performance significantly as E_c/N_o increases.

5. Conclusion

The stage 3/2 decoding scheme originally suggested for a noise-free channel is modified for a Rayleigh fading channel. The performance of a FFHMA/MFSK system with the MMDS was evaluated over a Rayleigh fading channel. When $E_b/N_o = 30$ dB, the simulation results show that the modified stage 3/2 decoding scheme increases the spectral efficiency by 11% compared to the modified stage 1 decoding scheme at $P_b = 10^{-3}$. If the modified stage 3/2 decoding scheme is used, E_b/N_o can be saved about 5 dB at $P_b = 10^{-3}$ without requiring large increase in complexity compared to the modified stage 1 decoding scheme.

Further, the performance of the MMDS is compared with that of the relatively simple DCMs. The OS-NED DCM and the MMDS have better performance than other methods. The modified stage 3/2 decoding scheme shows the best performance, and improves the BER performance significantly as E_c/N_o increases.

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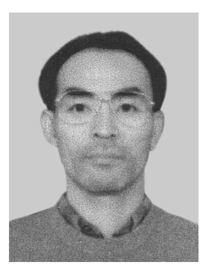

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