

## THREE-DIMENSIONAL MESH SIMPLIFICATION USING NORMAL VARIATION ERROR METRIC AND SUBDIVIDED EDGE CLASSIFICATION

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### ABSTRACT

In order to transmit or store three-dimensional (3-D) mesh models efficiently, we need an algorithm to simplify them. Although the quadric error metric (QEM) provides fast and accurate geometric simplification of 3-D mesh models, it cannot capture discontinuities efficiently. Recently, a new QEM based on subdivided edge classification has been proposed to handle this problem; thus, it can capture discontinuities, but it degrades reconstruction quality slightly. In this paper, we propose a novel mesh simplification algorithm, where we employ a normal variation error metric instead of QEM to resolve the quality degradation issue and we include subdivided edge classification to preserve discontinuities. We have tested the proposed algorithm with various 3-D VRML models. Simulation results show that the proposed algorithm provides good approximations while maintaining discontinuities; therefore, it can be used for 3-D mesh models having discontinuities.

### 1. INTRODUCTION

Recently, 3-D mesh models are used in various multimedia applications. Since 3-D models are expensive to render, transmit, and store due to a large amount of information, various mesh simplification algorithms have been proposed to address this problem by reducing the size of 3-D models.

As one of the conventional algorithms, the quadric error metric (QEM) provides fast and accurate geometric simplification of 3-D mesh models [1,2]. However, it cannot capture object discontinuities efficiently. In order to overcome this problem, we proposed an enhanced QEM based on subdivided edge classification [3,4].

Although the enhanced QEM can capture discontinuities efficiently, it causes quality degradation in surfaces of the 3-D model. The main reason for this problem is that it places more weights on types of edges than the relationship with neighboring primitives. In other words, it examines the edge type more seriously than how much the edge contraction changes the overall appearance, such as the surface curvature. Therefore, we need to improve the enhanced QEM algorithm, focusing on good quality approximations while preserving discontinuities.

In this paper, we propose a novel mesh simplification algorithm that is based on a normal variation error metric and subdivided edge classification in order to improve surface quality. We employ the normal variation error metric instead of QEM to measure the amount of geometric changes in the model as a result of edge contraction. We produce good approximations that remain faithful to the original 3-D model.

### 2. PROPOSED ALGORITHM

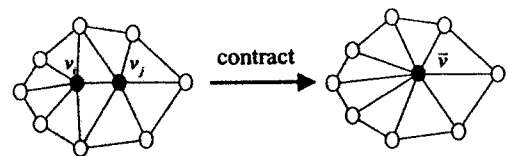
The proposed algorithm consists of three main components: iterative edge contraction, normal variation error metric, and subdivided edge classification.

#### 2.1 Iterative Edge Contraction

The simplification algorithm is based on the iterative contraction operation of edges. We start with the original mesh model and remove vertices and faces from the 3-D model iteratively until satisfying the given condition, namely, the number of faces. Each iteration involves a single atomic operation, namely, edge contraction.

By edge contraction, denoted by  $(v_i, v_j) \rightarrow \bar{v}$ , we modify the surface of the 3-D mesh model in the following three steps:

- STEP 1: move the vertices  $v_i$  and  $v_j$  to  $\bar{v}$ .
- STEP 2: replace all occurrences of  $v_j$  with  $v_i$ .
- STEP 3: remove  $v_j$  and all faces that become degenerated.



(a) Before contraction      (b) After contraction

Figure 1. Edge contraction

In the example illustrated in Figure 1, we delete one vertex and two faces from the mesh model.

## 2.2 Normal Variation Error Metric

How to measure the cost of a contraction is an important issue. Since we are concerned with producing simplified models that have similar appearance to the original model, the cost of the contraction should reflect how much that contraction changes the surface. In order to measure the amount of geometric change introduced into the model by the single edge contraction, we propose a new cost metric, the normal variation error metric. It was designed to reflect a correlation with neighboring primitives.

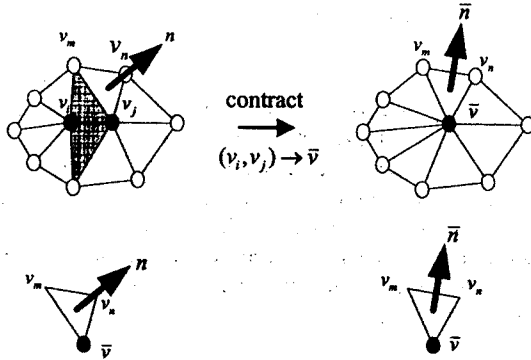


Figure 2. Normal variation by edge contraction

Figure 2 shows normal changes by edge contraction,  $e = (v_i, v_j) \rightarrow \bar{v}$ . If vertex  $v_i$  and vertex  $v_j$  are contracted to  $\bar{v}$ , faces having  $v_i$  or  $v_j$  as their primitive vertices come to be changed their face normal vectors. Therefore, we can assume the amounts of appearance changes of the model as the amounts of face normal variations.

For each vertex of the 3-D model, we can associate a set of triangular planes that meet at the vertex. With respect to this set, we define an error measure for edge contraction as the sum of the face normal variation to the plane set, which is multiplied by the estimated weight as follows:

$$Error = weight \times \sum_{planes(e)} (1 - n \cdot \bar{n}) \quad (1)$$

$$weight = const\_weight \times (1 \times IsInterior + IsComplex \times Complex\_Weight + IsBoundary \times Boundary\_Weight + IsBoundaryIncident \times Boundary\_Incident\_Weight) \quad (2)$$

where  $n$  and  $\bar{n}$  represent surface normal vectors before contraction and after contraction, respectively. Operator  $\cdot$  denotes the inner product.  $IsInterior$ ,  $IsComplex$ ,  $IsBoundary$ , and  $IsBoundaryIncident$  are binary Boolean variables. In other words, they can take TRUE(1) or FALSE(0). For example, if a given edge is a complex edge,  $IsComplex$  is set to TRUE. The values of  $Complex\_Weight$ ,  $Boundary\_Weight$ , and  $Boundary\_Incident\_Weight$  are defined in Section 2.3.2, and  $const\_weight$  is set by the user.

For the contraction operation of the edge  $(v_i, v_j)$ , we need to choose the target position  $\bar{v}$ . In the proposed algorithm, we simply select either  $v_i$ ,  $v_j$ , or  $(v_i + v_j)/2$ , which produces the smallest error that is defined in Eq. (1).

According to the target vertex position  $\bar{v}$ , there are three types of computation:

CASE 1: If  $\bar{v} = v_i$ , the highlighted triangular planes remain unchanged. However, the face normals of dotted triangular planes in Figure 2 will be changed. Therefore, we can compute normal variations of a plane set for the vertex  $v_j$  only.

CASE 2: If  $\bar{v} = v_j$ , the dotted triangular planes remain unchanged. However, the face normals of the highlighted triangular planes will be changed. Therefore, we can compute the normal variations of a plane set for the vertex  $v_i$  only.

CASE 3: If  $\bar{v} = (v_i + v_j)/2$ , the face normals of both dotted and highlighted triangular planes will be changed. Therefore, we can compute normal variations of the union plane set for the vertex  $v_i$  and vertex  $v_j$ .

Using the above properties, we can reduce required computation complexity compared to the QEM algorithm for CASE 1, and CASE 2.

## 2.3 Subdivided Edge Classification

Discontinuities in the 3-D model, such as creases, open boundaries, and borders between differently colored regions, are often among the most visually significant features. Therefore, preserving those features is critical for producing good approximations of the 3-D model.

In order to preserve discontinuities of the 3-D model, we employ the subdivided edge classification algorithm to preserve such object discontinuities effectively. Subdivided edge classification can be divided into two steps: edge classification and weight decision.

### 2.3.1 Edge Classification

In this step, we classify each edge into one of four different types: complex, boundary, boundary incident, and interior edge. Figure 3 shows the four different types of edges.

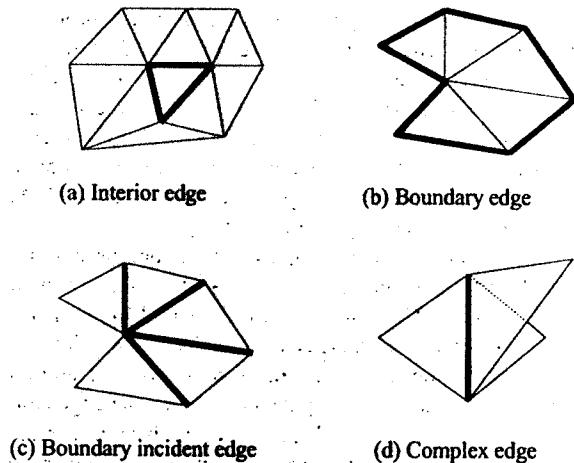


Figure 3. Edge types

As we can see in Figure 3, a boundary edge has exactly one incident triangle. A complex edge has three or more incident triangles. A boundary incident edge is not a boundary edge itself, but it has one or more incident boundary edges. An interior edge is neither a complex edge nor a boundary edge, and it has no incident boundary edge.

### 2.3.2 Weight Decision

All types of edges are candidates for deletion. However, we can assign an appropriate weight to each edge of the 3-D model according to its type and features.

#### A. Complex Edge

Since the shape changes of the 3-D mesh model due to contraction of complex edges could be severe, we give a large penalty on contracting the complex edges so that the contraction operation of complex edges can be conducted later relative to other types of edges. Hence, we define *Complex\_Weight* to be proportional to the number of incident triangles from the edge in the 3-D model.

$$\text{Complex\_Weight} = (\# \text{ of incident triangles} - 2) \quad (3)$$

#### B. Boundary Edge

The shape variation of the 3-D model becomes severe after contracting the boundary edges as well. Therefore, we must place a large penalty on contracting the boundary edges. Thus, we define *Boundary\_Weight* to be dependent on two dihedral angles,  $\theta_1$  and  $\theta_2$  by Eq. (4).

$$\text{Boundary\_Weight} = (1 + \max\{\cos \theta_1, \cos \theta_2\}) \quad (4)$$

#### C. Boundary Incident Edge

Contraction of boundary incident edges can yield substantial changes on the shape of the 3-D model. Since the shape change depends on the number of incident boundary edges, we define *Boundary\_Incident\_Weight* to be proportional to the number of incident boundary edges.

$$\text{Boundary\_Incident\_Weight} = (\# \text{ of incident boundary edges}) / 4 \quad (5)$$

#### D. Interior Edge

Since the shape changes of the 3-D model after contracting the interior edges is milder relative to any other types of edges, we assign no penalty on contracting the interior edges.

## 3. EXPERIMENTAL RESULTS

### 3.1 Model Simplification

As shown in Figure 4(a), the "SHARK" model is a non-manifold surface that has 468 vertices and 734 faces. It has 186 open discontinuity edges and 6 complex edges. In Figure 4(b), Figure 4(c), and Figure 4(d), we show three approximations of 250 faces, i.e., only 34% of the faces of the original model.

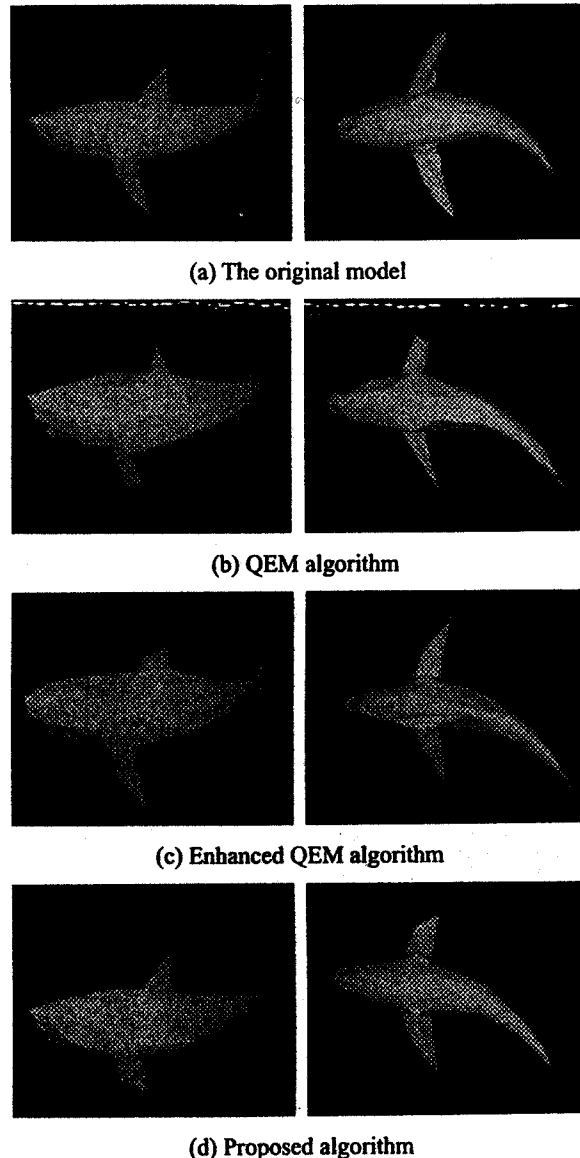
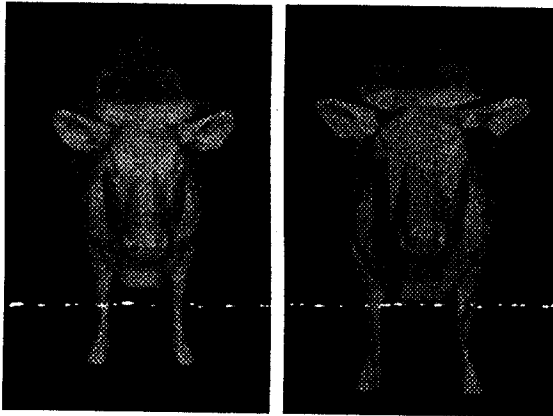


Figure 4. SHARK model

Figure 4(b) and Figure 4(c) show simplified models generated by the QEM [1] and the enhanced QEM [4] schemes, respectively. From Figure 4(c), we notice that the enhanced QEM using subdivided edge classification preserves discontinuities well especially in the tail fin of the SHARK model. However, the surface quality is degraded compared to Figure 4(b). Figure 4(d) shows an approximation generated by our proposed algorithm.

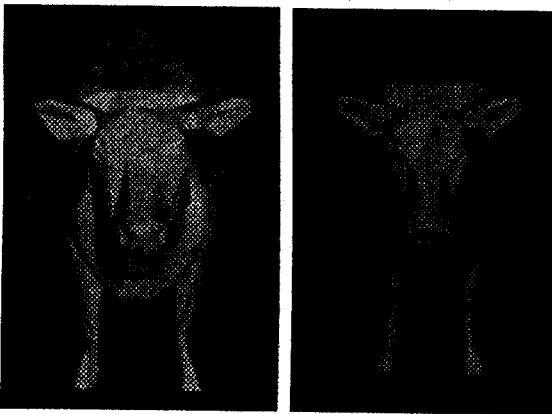
In the right side of Figure 4, another view of the same model is presented to show improvement of surface quality. Especially, the belly and the ventral fins assure that the proposed algorithm generates more accurate approximation compared to other methods.

In order to evaluate performance with 3-D models of no discontinuities, we have applied the proposed algorithm to the COW model. Figure 5(a) shows the original COW model that has 2904 vertices and 5804 faces. It does not have any boundary, complex and boundary incident edges. There are only interior edges.



(a) Original COW model

(b) QEM algorithm  
[Distortion Error: 0.85]



(c) Enhanced QEM algorithm  
[Distortion Error: 0.85]

(d) Proposed algorithm  
[Distortion Error: 1.11]

Figure 5. COW model

In Figure 5(b), Figure 5(c), and Figure 5(d), we show approximations with 994 faces, i.e., only 17% of the faces of the original model. Figure 5(b) is generated by QEM, and Figure 5(c) is generated by the enhanced QEM scheme using subdivided edge classification. For 3-D models that have only interior edges, the same quadric error metric is used both in QEM and enhanced QEM using subdivided edge classification. In other words, the QEM and the enhanced QEM algorithms using subdivided edge classification generate the same approximations, as shown in Figure 5(b) and Figure 5(c).

However, the proposed algorithm employs the normal variation error metric instead of quadric error metric. Therefore, even for the 3-D model with only interior edges, the proposed algorithm produces different approximations from the QEM method. Figure 5(d) demonstrates a simplified model generated by the proposed algorithm with the normal variation error metric. The result in Figure 5(d) is quite different from the approximation in Figure 5(b). By comparing these approximations, we notice that the surface quality of the approximation by the proposed algorithm is slightly degraded compared to the one by the QEM algorithm in Figure 5(b) in terms of the Hausdorff distortion error measure [4]. This result indicates that the normal variation error metric cannot be a general solution by itself,

since it is designed to reduce quality degradation in the subdivided edge classification algorithm.

## 4. CONCLUSIONS

In this paper, we propose a novel simplification algorithm for 3-D mesh models based on the normal variation error metric and subdivided edge classification. Subdivided edge classification is used to maintain discontinuities, and the normal variation error metric is used to resolve surface quality degradation in the enhanced QEM using subdivided edge classification. The proposed normal variation error metric defines an error for edge contraction as the sum of the face normal changes so as to reflect the correlation with neighboring primitives. Simulation results demonstrate that the proposed algorithm can be applied to 3-D mesh models with discontinuities in order to achieve good quality approximations and maintain discontinuities.

As a future work, we can extend the normal variation error metric both to 3-D mesh models of no discontinuities and to 3-D models with appearance attributes.

## ACKNOWLEDGEMENT

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