Three-Dimensional Mesh Simplification using Normal Variation Error Metric and Modified Subdivided Edge Classification

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ABSTRACT

In order to transmit or store three-dimensional (3-D) mesh models efficiently, we need to simplify them. Although the quadric error metric (QEM) provides fast and accurate geometric simplification of 3-D mesh models, it cannot capture discontinuities faithfully. Recently, an enhanced QEM based on subdivided edge classification has been proposed to handle this problem. Although it can capture discontinuities well, it has slight degradation in the reconstruction quality. In this paper, we propose a novel mesh simplification algorithm where we employ a normal variation error metric, instead of QEM, to resolve the quality degradation issue. We also modify the subdivided edge classification algorithm to be cooperative with the normal variation error metric while preserving discontinuities. We have tested the proposed algorithm with various 3-D VRML models. Simulation results demonstrate that the proposed algorithm provides good approximations while maintaining discontinuities well.

Keywords: Mesh simplification, error metric, normal variation, edge classification, VRML

1. INTRODUCTION

Recently, 3-D mesh models are used in various multimedia applications. Since 3-D models are expensive to render, transmit, and store due to a large amount of information, various mesh simplification algorithms have been proposed to address this problem by reducing the size of 3-D models.

As one of the conventional algorithms, the quadric error metric (QEM) provides fast and accurate geometric simplification of 3-D mesh models[1,2]. However, it cannot capture object discontinuities efficiently. In order to overcome this problem, we proposed an enhanced QEM based on subdivided edge classification[3,4].

Although the enhanced QEM can capture discontinuities well, it causes quality degradation in surfaces of the simplified 3-D model. The main reason for this problem is that it places more weights on types of edges than the relationship with neighboring primitives. In other words, it examines the edge type more seriously than how much the edge contraction changes the overall appearance, such as the surface curvature. Therefore, we need to improve the enhanced QEM algorithm, focusing on good quality approximations while preserving discontinuities.

In this paper, we propose a novel mesh simplification algorithm that is based on a normal variation error metric and a modified subdivided edge classification in order to improve surface quality while preserving discontinuities. We employ the normal variation error metric instead of QEM to measure the amount of geometric changes induced into the model as a result of an edge contraction. It assumes the amounts of appearance changes of the model as the amounts of face normal variations to reflect a correlation with neighboring primitives. Furthermore, we modified subdivided edge classification adopted in the enhanced QEM algorithm to be operable with normal variation error metric for preserving discontinuities. Former subdivided edge classification comes to have some problems by adopting normal variation error metric. To address these problems, we take into diverse circumstances of an edge contraction operation. Thus, we classify edges into more various types and we assign a proper weight according to their types and features in the 3-D model. Using these key methodologies, we can produce good approximations that remain faithful to the original 3-D model.

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2. PROPOSED ALGORITHM

The proposed algorithm consists of three main components: iterative edge contraction, normal variation error metric, and modified subdivided edge classification.

2.1 Iterative edge contraction

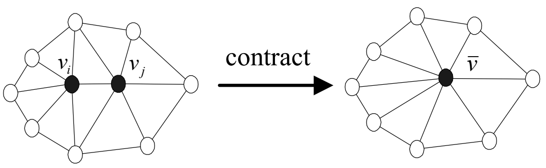
Proposed simplification algorithm is based on the iterative contraction operation of edges[1,2]. We start with the original mesh model and remove vertices and faces from the 3-D model iteratively until satisfying the given condition, namely, the number of faces. Each iteration involves a single atomic operation, namely, edge contraction.

By edge contraction, denoted by $(v_i, v_j) \rightarrow \bar{v}$, we modify the surface of the 3-D mesh model in the following three steps:

STEP 1: move the vertices v_i and v_j to $\bar{v}$.

STEP 2: replace all occurrences of v_j with v_i.

STEP 3: remove v_j and all faces that become degenerated.



(a) Before contraction (b) After contraction

Figure 1: Edge contraction

In the example illustrated in Figure 1, we delete one vertex and two faces from the mesh model.

2.2 Normal variation error metric

How to measure the cost of a contraction is an important issue. Since we are concerned with producing simplified models that have similar appearance to the original model, the cost of the contraction should reflect how much that contraction changes the surface. In order to measure the amount of geometric change introduced into the model by the single edge contraction, we propose a new cost metric, the normal variation error metric[6]. It was designed to reflect a correlation with neighboring primitives.

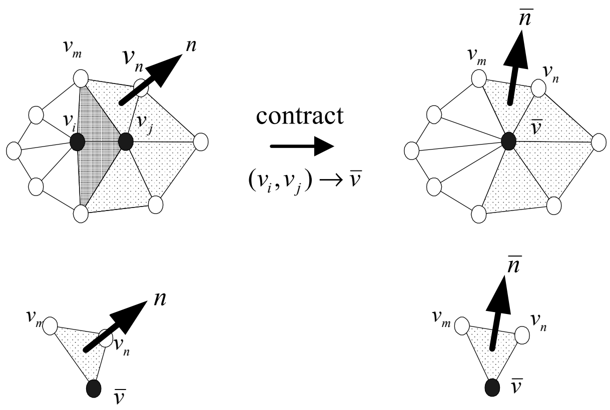


Figure 2: Normal variation by an edge contraction

Figure 2 shows normal changes by edge contraction, $e=(v_i,v_j)\rightarrow\bar{v}$. If vertex v_i and vertex v_j are contracted to $\bar{v}$, faces having v_i or v_j as their primitive vertices come to be changed their face normal vectors. Therefore, we can assume the amounts of appearance changes of the model as the amounts of face normal variations. For reference, degenerated faces having $e=(v_i,v_j)$ as their primitive edge are ignored when calculating amounts of normal variations because their normal changed pairs are removed from 3-D model by an edge contraction.

For each vertex of the 3-D model, we can associate a set of triangular planes that meet at the vertex. With respect to this set, we define an error measure for edge contraction as the sum of the face normal variation to the plane set, which is multiplied by the estimated weight as follows:

$$Error = weight \times \sum_{planes(e)} (1 - n \bullet \bar{n}) \tag{1}$$

$$\begin{aligned} weight = {} & const\_weight \times (1 \times IsInterior + IsComplex \\ & \times Complex\_Weight + IsBoundary \times \\ & \times Boundary\_Weight + IsBoundary\ Incident \\ & \times Boundary\_Incident\_Weight \end{aligned} \tag{2}$$

where n and $\bar{n}$ represent surface normal vectors before contraction and after contraction, respectively. Operator $\bullet$ denotes the inner product. *IsInterior*, *IsComplex*, *IsBoundary*, and *IsBoundaryIncident* are binary Boolean variables. In other words, they can take TRUE(1) or FALSE(0). For example, if a given edge is a complex edge, *IsComplex* is set to TRUE. The values of *Complex_Weight*, *Boundary_Weight*, and *Boundary_Incident_Weight* are defined in Section 2.3.2, and *const_weight* is set by the user.

For the contraction operation of the edge (v_i,v_j), we need to choose the target position $\bar{v}$. In the proposed algorithm, we simply select either v_i, v_j, or $(v_i+v_j)/2$, which produces the smallest error that is defined in Eq. (1). According to the target vertex position $\bar{v}$, there are three types of computation:

CASE 1: If $\bar{v}=v_i$, the highlighted triangular planes remain unchanged. However, the face normals of dotted triangular planes in Figure 2 will be changed. Therefore, we can compute normal variations of a plane set for the vertex v_j only.

CASE 2: If $\bar{v}=v_j$, the dotted triangular planes remain unchanged. However, the face normals of the highlighted triangular planes will be changed. Therefore, we can compute the normal variations of a plane set for the vertex v_i only.

CASE 3: If $\bar{v}=(v_i+v_j)/2$, the face normals of both dotted and highlighted triangular planes will be changed. Therefore, we can compute normal variations of the union plane set for the vertex v_i and vertex v_j.

Using the above properties, we can reduce required computation complexity compared to the QEM algorithm for CASE 1 and CASE 2.

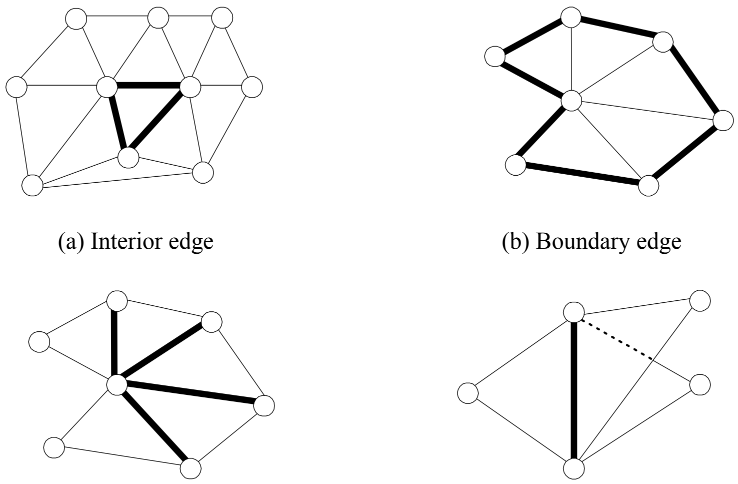
2.3 Modified subdivided edge classification

Discontinuities in the 3-D model, such as creases, open boundaries, and borders between differently colored regions, are often among the most visually significant features. Therefore, preserving those features is critical for producing good approximations of the 3-D model.

In order to preserve discontinuities of the 3-D model, we employ the modified subdivided edge classification algorithm. Subdivided edge classification adopted in the enhanced QEM algorithm comes to have some problems by adopting normal variation error metric. To address these problems, we classify edges into more various types and we assign a proper weight according to their types and features in the 3-D model. Modified subdivided edge classification can be divided into two steps: edge classification and weight decision.

2.3.1 Edge classification

In this step, we classify each edge into one of four representative types: complex, boundary, boundary incident, and interior edge. Figure 3 shows the four different types of edges. As we can see in Figure 3, a boundary edge has exactly one incident triangle. A complex edge has three or more incident triangles. A boundary incident edge is not a boundary edge itself, but it has one or more incident boundary edges. An interior edge is neither a complex edge nor a boundary edge, and it has no incident boundary edge.

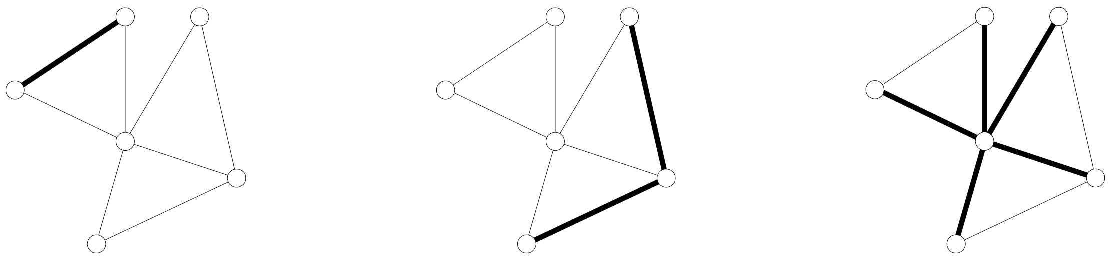


(a) Interior edge (b) Boundary edge

(c) Boundary incident edge (d) Complex edge

Figure 3: Edge types

Furthermore, modified subdivided edge classification categorize the boundary edge into four different subtypes according to their topological and geometrical characteristics: B1 boundary, B2 boundary, B2 coplanar boundary, and BM boundary edge. Figure 4 shows the four different subtypes of the boundary edge.



(a) B1 boundary edge (b) B2 boundary/B2 coplanar boundary edge (3) BM boundary edge

Figure 4: Subtypes of the boundary edge

B1 boundary, B2 boundary, and BM boundary edge are distinguished by topological characteristics. B2 boundary and B2 coplanar boundary edge are classified by geometrical characteristics. B1 boundary edge has only one neighborhood face having their primitive vertex as given boundary edge's primitive vertex. B2 boundary and B2 coplanar edge have two neighborhood faces. If edge's two neighborhood faces are placed on same plane, we define it as B2 coplanar boundary. If not, we define it as B2 boundary edge. BM boundary edge has three or more neighborhood faces.

2.3.2 Weight decision

All types of edges are candidates for deletion. However, we can assign an appropriate weight to each edge of the 3-D model according to its type and features.

A. Complex edge

Since the shape changes of the 3-D mesh model due to contraction of complex edges could be severe, we give a large penalty on contracting the complex edges so that the contraction operation of complex edges can be conducted later relative to other types of edges. Hence, we define *Complex_Weight* to be proportional to the number of incident triangles from the edge in the 3-D model.

$$Complex\_Weight = (\#\,of\ incident\ triangles - 2) \tag{3}$$

At the worst case, if we contract the given complex edge, all neighborhood triangles are degenerated from the 3-D mesh model. As we mentioned before, normal variation error metric does not take into degenerated faces when calculating the error. In this case, even though the severest changes are introduced by the complex edge contraction operation, estimated error is zero. To deal with this problem, if calculated error on the basis of normal changes is smaller than one, we set final error as *weight* defined in Eq. (2). If it is equal to or greater than one, we calculate final error as Eq. (1):

$$\begin{aligned} &Error_{Complex\ edge} = weight; && if,\ \sum_{planes(e)}(1 - n \bullet \bar{n}) < 1 \\ &Error_{Complex\ edge} = weight \times \sum_{planes(e)}(1 - n \bullet \bar{n}); && otherwise \end{aligned} \tag{4}$$

B. Boundary edge

The shape variation of the 3-D model becomes severe after contracting the boundary edges as well. Therefore, we must place a large penalty on contracting the boundary edges. Thus, we define *Boundary_Weight* to be dependent on two dihedral angles, θ_1 and θ_2 by Eq. (4).

$$Boundary\_Weight = (1 + \max\{\cos\theta_1, \cos\theta_2\}) \tag{5}$$

As we stated above, B1 boundary edge has only one neighborhood face. Thus, after contracting the B1 boundary edge its neighborhood face is removed from the 3-D model, we cannot estimate the exact error using normal variation error metric. Furthermore, there is only one face, we cannot use dihedral angle property as shown in Eq. (5). To overcome these problems, we utilize a face area as an error metric instead of the normal variation to reflect the contribution of its neighborhood face into the 3-D model, exceptionally. Therefore, we set the final estimated error as the value of its neighborhood face area divided by maximum face area within the given 3-D model:

$$Error_{B1\ boundary\ edge} = \frac{area\ of\ neighborhood\ face}{maximum\ face\ area} \times const\_weight \tag{6}$$

B2 boundary edge and B2 coplanar boundary edge have two neighborhood faces. After contracting B2 coplanar boundary edge, one of its neighborhood faces is removed and normal of the remained face is unchanged. Hence, we just get useless zero value errors on the basis of normal variation error metric. To address this problem, we set the final estimated error as the area change between the summed area of original two neighborhood faces and the area of the remained face divided by maximum face area within the given 3-D model with applying the dihedral angle penalties defined in Eq. (5).

After contracting B2 boundary edge, one of its neighborhood faces is degenerated as well. However, normal of the remained face comes to be changed. According to the target vertex position $\bar{v}$, we can get three estimated errors concerning the given edge contraction. One of them has zero value error because of CASE 1 and CASE 2 property described in Section 2.2. We only take into two errors having non-zero value errors concerning the given edge contraction, then we apply the dihedral angle penalties, Boundary Weight, defined in Eq. (5) to them. Namely, we ignore this zero value error because it cannot assure the global minimum error.

There is no delicate situation after contracting the BM boundary edge. Therefore, we simply estimate errors using Eq. (1) with no exceptional condition.

C. Boundary incident edge

Contraction of boundary incident edges can yield substantial changes on the shape of the 3-D model. Since the shape change depends on the number of incident boundary edges, we define *Boundary_Incident_Weight* to be proportional to the number of incident boundary edges.

$$\begin{aligned} & Boundary\ \_Incident\ \_Weight \\ & = (\#\ of\ \ incident\ \ boundary\ \ edges)/40 \end{aligned} \tag{7}$$

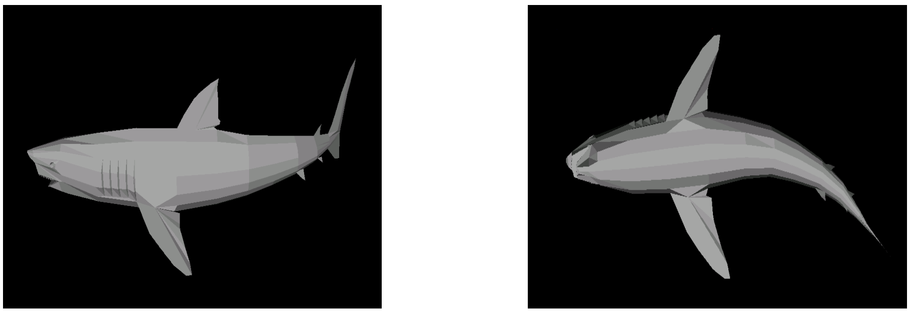
D. Interior edge

Since the shape changes of the 3-D model after contracting the interior edges is milder relative to any other types of edges, we assign no penalty on contracting the interior edges.

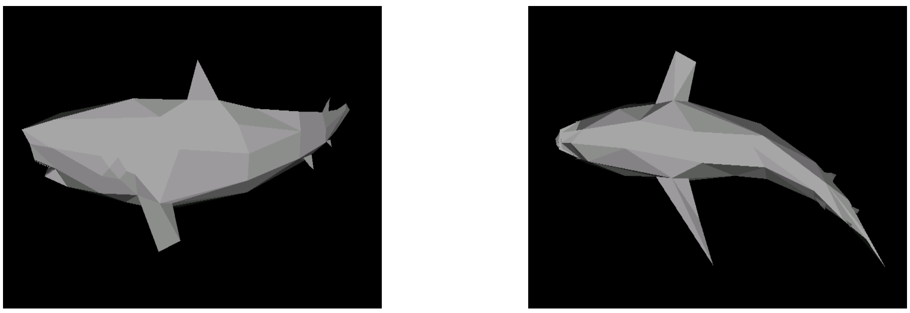
3. EXPERIMENTAL RESULTS

3.1 MODEL SIMPLIFICATION

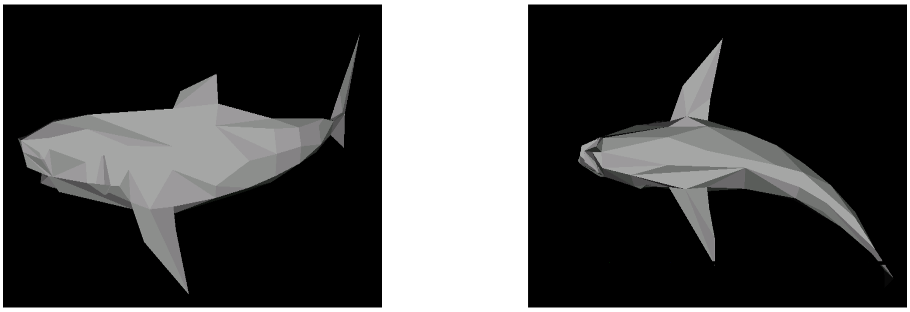
As shown in Figure 5(a), the "SHARK" model is a non -manifold surface that has 468 vertices and 734 faces. It has 186 open discontinuity edges and 6 complex edges. In Figure 5(b), Figure 5(c), and Figure 5(d), we show three approximations of 250 faces, i.e., only 34% of the faces of the original model.



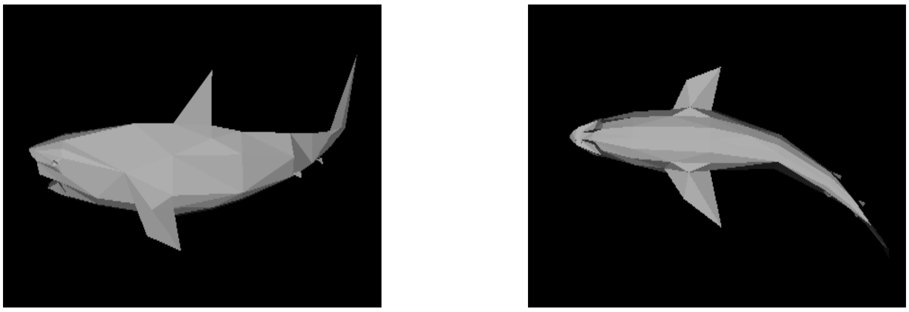
(a) Original model



(b) QEM algorithm



(c) Enhanced QEM algorithm

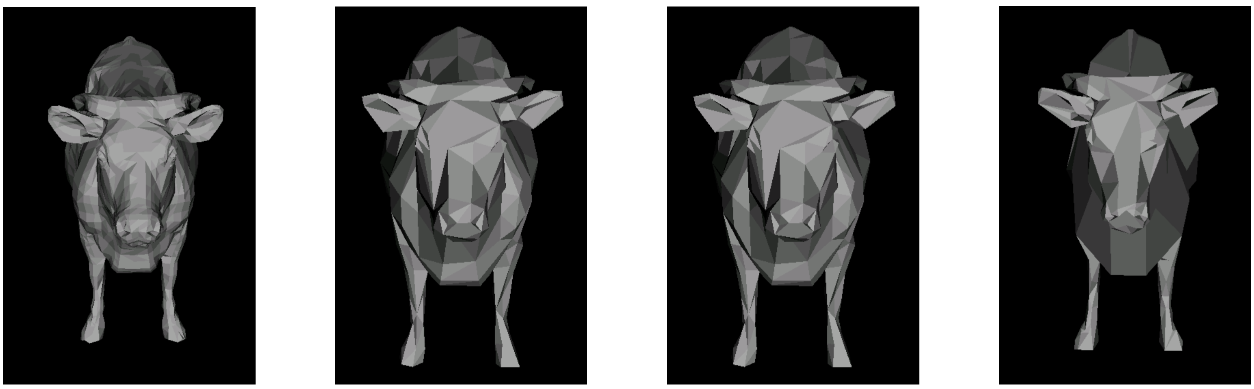


(d) Proposed algorithm

Figure 5: SHARK model

Figure 5(b) and Figure 5(c) show simplified models generated by the QEM[1] and the enhanced QEM[4] schemes, respectively. From Figure 5(c), we notice that the enhanced QEM using subdivided edge classification preserves discontinuities well especially in the tail fin of the SHARK model. However, the surface quality is degraded compared to Figure 5(b). Figure 5(d) shows an approximation generated by our proposed algorithm.

In the right side of Figure 5, another view of the same model is presented to show improvement of surface quality. Especially, the belly and the ventral fins assure that the proposed algorithm generates more accurate approximation compared to other methods. Moreover, modified subdivided edge classification maintains discontinuities faithfully compared to the subdivided edge classification.



(a) Original model (b) QEM algorithm [Distortion Error: 0.85] (c) Enhanced QEM algorithm [Distortion Error: 0.85] (d) Proposed algorithm [Distortion Error: 1.11]

Figure 6: COW model

In order to evaluate performance with 3-D models of no discontinuities, we have applied the proposed algorithm to the COW model. Figure 6(a) shows the original COW model that has 2904 vertices and 5804 faces. It does not have any boundary, complex and boundary incident edges. There are only interior edges.

In Figure 6(b), Figure 6(c), and Figure 6(d), we show approximations with 994 faces, i.e., only 17% of the faces of the original model. Figure 6(b) is generated by QEM, and Figure 6(c) is generated by the enhanced QEM scheme using subdivided edge classification. For 3-D models that have only interior edges, the same quadric error metric is used both in QEM and enhanced QEM using subdivided edge classification. In other words, the QEM and the enhanced QEM algorithms using subdivided edge classification generate the same approximations, as shown in Figure 6(b) and Figure 6(c).

However, the proposed algorithm employs the normal variation error metric instead of quadric error metric. Therefore, even for the 3-D model with only interior edges, the proposed algorithm produces different approximations from the QEM method. Figure 6(d) demonstrates a simplified model generated by the proposed algorithm with the normal variation error metric. The result in Figure 6(d) is quite different from the approximation in Figure 5(b). By comparing these approximations, we notice that the surface quality of the approximation by the proposed algorithm is slightly degraded compared to the one by the QEM algorithm in Figure 6(b) in terms of the Hausdorff distortion error measure[4]. This result indicates that the normal variation error metric cannot be a general solution by itself, since it is designed to reduce quality degradation in the subdivided edge classification algorithm.

4. CONCLUSIONS

In this paper, we have proposed a new simplification algorithm for 3-D mesh models based on the normal variation error metric and modified subdivided edge classification. We have defined a modified subdivided edge classification method to maintain discontinuities, and the normal variation error metric to resolve surface quality degradation in the enhanced QEM using the subdivided edge classification method. The proposed normal variation error metric defines an error for edge contraction as the sum of the face normal changes so as to reflect the correlation with neighboring primitives. In addition, the modified subdivided edge classification method is slightly improved to be cooperative with the normal variation error metric. Simulation results demonstrate that the proposed algorithm can be applied to 3-D mesh models having discontinuities to achieve good approximation quality and maintain discontinuities.

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