ANALYSIS OF QUANTIZATION WATERMARKING IN THE WAVELET TRANSFORM DOMAIN

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ABSTRACT

Quantization watermarking is a method of embedding hidden copyright information based on dithered quantization. Such a scheme is only practical for watermarking applications, where the original signal is available to the detector as for a fingerprinting purpose. The aim of this paper is to analyze the quantization watermarking in the wavelet transform or equivalently in filter bank domain. We involve the nonlinear effect of dithered quantization in the filter bank analysis using the time-domain formulation of the filter bank. We derive a general and compact form for distortion in the host image due to the encoding and embedding process. The formulation can be simplified and optimized for different filter banks and dither signals. We provide some supporting experiments for the analysis.

1 INTRODUCTION

Digital watermarking is an approach for copyright protection by embedding the user signature or other copyright information directly in the host data. In quantization watermarking, the signature information is used as a dither signal in the process of quantizing the host image [1,2]. This method is especially useful for those applications where compression of the host image with embedding the secret information is jointly implemented.

Using theoretical results of dithered quantization [2], Eggers and Girod derived a mathematical analysis of quantization watermarking based on probability density function (PDF) of the host and signature signals [1]. The aim of this paper is to extend their results for quantization watermarking of wavelet coefficients of the host signal. Since quantization is a non-linear process, we choose the time-domain framework introduced by Nayebi et. al. [3] in order to analyze quantization watermarking in the wavelet domain. We derive a statistical form of this formulation that could be used to analyse the distortion in the host image due to embedding the signature information and encoding the subbands [4,5].

In the following sections, after we provide a brief explanation on dithered quantization, we explain the modification of time-domain formulation of filter bank for the quantization watermarking effect and its simplification.

2 DITHERED QUANTIZATION

Figure 1 shows the non-subtractive dithering scheme. The host signal x(n) is the main input to quantizer, and the signature signal d(n) is the added dither.

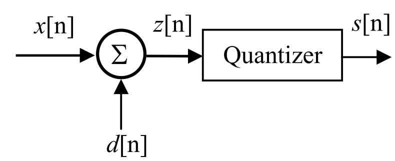

Fig. 1. Non-Subtractive dithered quantization

The quantization error in the uniform scalar quantizer is only limited to (-Δ/2 , Δ/2), where Δ is the quantization step size. The aim of dithering in the classical signal coding system is to improve perceptual quality of the reconstructed signal by changing the quantization error spectrum [2]. The characteristic function of the input z(n) to the quantizer can be written by

$$M_z(ju) = M_x(ju)M_d(ju) \qquad (1)$$

where M_x and M_d are the characteristic functions of the input signal and the dither respectively. It can be proved [1] that the characteristic function of the quantization noise e(n) is represented in terms of the input signal and the dither signal characteristic functions [1]

$$M_e(ju) = \sum_{b=-\infty}^{\infty} M_z(j\frac{2\pi b}{\Delta}) \cdot Sinc(\frac{\Delta}{2}(u + 2\pi b/\Delta)) \qquad (2)$$

By substituting $M_z(u)$ from Eq. (2) by Eq.(1), we have

$$M_e(ju) = \sum_{b=-\infty}^{\infty} M_x(j\frac{2\pi b}{\Delta}) \cdot M_d(j\frac{2\pi b}{\Delta}) \cdot Sinc(\frac{\Delta}{2}(u + 2\pi b/\Delta)) \qquad (3)$$

Using the characteristic function, the energy of quantization noise can be computed by

$$E[e^2] = -\frac{d^2}{du^2} M_e(u)\Big|_{u=0} = \frac{\Delta^2}{12} + \sum_{\substack{b=-\infty \\ b\neq 0}}^{\infty} \frac{(-1)^b}{2(\pi b/\Delta)^2} M_x(\frac{j2\pi b}{\Delta}) . M_d(\frac{j2\pi b}{\Delta}) \qquad (4)$$

The cross-correlation of the quantization error and the dither signal is

$$E[ed] = \sum_{\substack{b=-\infty \\ b\neq 0}}^{\infty} \frac{(-1)^b}{2\pi b/\Delta} M_x(\frac{j2\pi b}{\Delta}) . \mathrm{Im}\{M_d^1(\frac{j2\pi b}{\Delta})\} \qquad (5)$$

where

$$M_d^1(\frac{j2\pi b}{\Delta}) = \frac{d}{du}\{M_d(\frac{j2\pi b}{\Delta})\}\Big|_{u=0} \qquad (6)$$

In most cases, the signal PDF and its characteristic function are even functions, therefore the summation in Eq. (5) can be written as

$$E[ed] = \sum_{b=1}^{\infty} \frac{(-1)^b}{\pi b/\Delta} M_x(\frac{j2\pi b}{\Delta}).\operatorname{Im}\{M_d^1(\frac{j2\pi b}{\Delta})\} \qquad (7)$$

If we want to check only the existence or absence of the watermark signal, we can use a correlation detector. In this case the output of the correlation detector for the two case, of existence of the watermark (U_1) and its absence (U_0) could be derived by [1]:

$$U_1 = \frac{E[(e(n)d(n)]}{\sigma_d^2} + 1 \qquad (8)$$

$$U_0 = \frac{E[(e(n)d(n)]}{\sigma_d^2} \qquad (9)$$

which implies that the absolute value of the normalized cross-correlation between the quantization error and the dither signal ($E[ed]/\sigma_d^2$) should be as small as possible. At the same time, to reduce the visual distortion, we should minimize the quantization error.

3 TIME-DOMAIN FORMULATION

3.1 General Formulation

In this section we derive a formulation that shows the effect of the added watermark in the wavelet transform domain. The distortion could be calculated based on reconstruction error in the corresponding filter bank.

Figure 2 shows the block diagram of the conventional filter bank structure. Figure 3 shows the filter bank with dithered quantizers. As depicted in Figure 3, we model the effect of quantization as additive, but not necessarily un-correlated signals, $F_i(m)$. The dither signal in each channel is represented by $D_i(m)$. The output signal $\hat{x}(n)$ is synthesized from the quantized subband signals. In order to analyze and design the subband coder, we only consider a uniform M band filter bank with filters of length N and the overall delay of Δ samples, such that L=N / M is an integer. This result can be easily extended to nonuniform or multidimensional subbands. The relationship between input and output of the system, in the time-domain can be expressed as

$$\hat{x}(Mm) = s^T(A^T x_I(Mm) + F_q(m) + D(m)) \qquad (10)$$

where input and output vectors of length **M** and **I** are

$$\hat{\boldsymbol{x}}_M(\boldsymbol{n}) = [\hat{\boldsymbol{x}}(\boldsymbol{n}+\boldsymbol{M}-1), \hat{\boldsymbol{x}}(\boldsymbol{n}+\boldsymbol{M}-2), \ldots, \hat{\boldsymbol{x}}(\boldsymbol{n})]^T \qquad (11)$$

$$\boldsymbol{x}_I(\boldsymbol{n}) = [\boldsymbol{x}(\boldsymbol{n}), \boldsymbol{x}(\boldsymbol{n}-1), \ldots, \boldsymbol{x}(\boldsymbol{n}-\boldsymbol{I}+1)]^T \qquad (12)$$

the parameter I is equal to ***2N-M*** , since the analysis and synthesis filter together create 2N delay, and the output is calculated in M points, therefore we need ***2N-M+1*** points of input. Note that, in Eq. (10) because of mixing the two terms of quantization and dithering, before up-samplers, the time index of the input and the output signal changed from ***m*** to ***mM*** . Finally, **A** is a block Toeplitz matrix of size **I** × **N** defined as

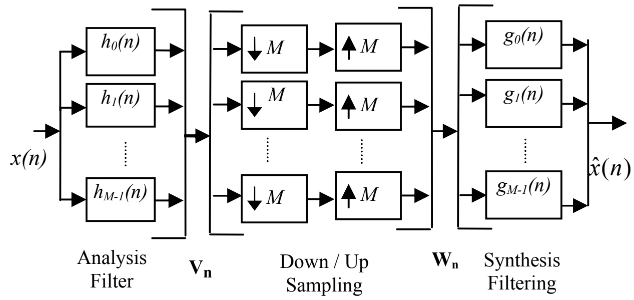

Fig. 2. Block Diagram of a Basic Filter Bank

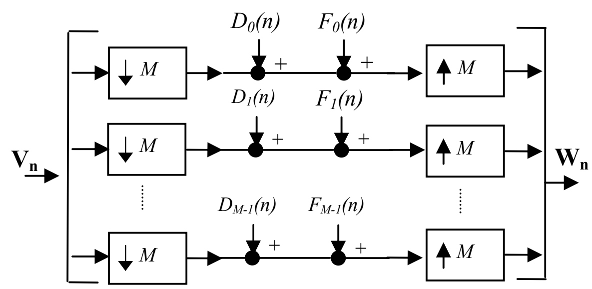

Fig. 3. Down/ Up sampling part of the subband coder with including the dithered quantizer

$$A(n) = \begin{bmatrix} [P^t] & 0 & .. & 0 \\ 0 & [P^T] & & \vdots \\ \vdots & 0 & \ddots & \vdots \\ & \vdots & \ddots & 0 \\ 0 & 0 & .. & [P^T] \end{bmatrix} \qquad (13)$$

where **P** is an **M** × **N** matrix whose **i**th row is comprised of the coefficients of the **i**th analysis filter, and **O** is an **M** × **M** zero matrix. The matrix ***s*** consists of the synthesis filter coefficients

$$s = \begin{bmatrix} g_0(0) & g_1(0) & \ldots & g_{M-1}(0) \\ g_0(1) & g_1(1) & \ldots & g_{M-1}(1) \\ \vdots & \vdots & \ddots & \vdots \\ g_0(N-1) & g_1(N-1) & .. & g_{M-1}(N-1) \end{bmatrix} \qquad (14)$$

and $g_i(j)$ denotes the **j** coefficient of the **i** synthesis filter. Finally, the vector $D(m)$ represents the dither signal, and $\boldsymbol{F_q}(\boldsymbol{m})$ the quantization noise:

$$F_q(m) = [q^T(m), q^T(m-1), \ldots, q^T(m-L+1)]^T \qquad (15)$$

$$D(m) = [d^T(m), d^T(m-1), \ldots, d^T(m-L+1)]^T \qquad (16)$$

where

$$d(m) = [d_0(m), d_1(m), \ldots, d_{M-1}(m)]^T \qquad (17)$$

$$q(m) = [q_0(m), q_1(m), \ldots, q_{M-1}(m)]^T \qquad (18)$$

are the dither and the quantization signal at time **m**.

Assuming that the filter bank is perfect reconstruction, the relationship between input and output of the filter bank can be written as

$$\hat{x}(Mm) = b^T x_I(Mm) \tag{19}$$

where the matrix **b** denotes one period of the ideal impulse response of the system. Therefore, using (10) and (20), one period of the output error can be written as

$$\begin{aligned} e &= \hat{x}(Mm) - x(Mm-\Delta) \\ &= (As-b)^T x_I(Mm) + s^T F_q(m) + s^T D(m) \end{aligned} \tag{20}$$

We use the mean square error of the output as a criterion for minimization assuming that the input signal can be modelled as zero-mean, wide sense stationary (WSS) sources

$$\bar{\sigma}_e^2 = \frac{1}{M} Trace[E\{ee^T\}] \tag{21}$$

where "Trace" denotes the sum of main matrix diagonal elements. Using (21), it can be shown that

$$\begin{aligned} \bar{\sigma}_e^2 &= \frac{1}{M} Trace[(As-b)^T R_{xx}(As-b)] + \\ & \frac{1}{M} Trace[s^T R_{qq} s] + \frac{2}{M} Trace[(As-b)^T R_{xq} s] + \\ & \frac{1}{M} Trace[s^t R_{dd} s] + \frac{2}{M} Trace[(As-b)^T R_{xd} s] + \\ & \frac{2}{M} Trace[s^T R_{qd} s] \end{aligned} \tag{22}$$

$\mathbf{R_{xx}}$, $\mathbf{R_{qq}}$ and $\mathbf{R_{dd}}$ are the input, the quantization noise and the dither signal autocorrelation matrices respectively, while $\mathbf{R_{xq}}$,$\mathbf{R_{xd}}$ and $\mathbf{R_{qd}}$ represents the cross-correlations between these signals.

Eq. (22) is the basic formulation that shows the distortion effect due to dither signal and quantization error. The optimization should be undertaken with addition of a Lagrangian cost with constraint of a fixed total bit-rate R_T

$$\sum_{k=0}^{M-1} R_k = R_T \tag{23}$$

where R_k denotes the bit-rate in subband k . In the encoding process, two situations are of particular interest.

- Subband signals are split into blocks and the number of bits used for a given block depends on the dynamic range of these signals.
- Entropy coding is performed in each subband. In this case the optimization is carried out under the constraint of a given entropy budget H_T .

For the uniform scalar quantization, which we use in quantization watermarking, R_k and Δ_k are related as $R_k = \log_2(d_k/\Delta_k)$, where d_k is the dynamic range of the signal and Δ_k is the quantization bin.

In order to maximize the correct watermark detection probability, for each subband quantizer, we should minimize the normalized absolute value of cross-correlation between the quantization noise and the watermark $|E[e_{q_i}.d_i]|/\sigma_{d_i}^2$, for each channel [1]. At the same time, we should minimize the total reconstruction error $\bar{\sigma}_e^2$ to reduce visible distortion

3.2 Simplification of Formulation

In the design of the wavelet-based quantization watermarking scheme, it sounds reasonable to simplify the design by selecting a perfect reconstruction filter bank (***As=b***) and optimize the system performance by a proper selection of quantizers and the variance of the added watermark signal to each subband. By selecting perfect reconstruction filters, we can simplify Eq. (22)

$$\begin{aligned} \bar{\sigma}_e^2 &= \frac{1}{M} Trace[s^T R_{qq} s] + \frac{1}{M} Trace[s^t R_{dd} s] \\ &+ \frac{2}{M} Trace[s^T R_{qd} s] \end{aligned} \tag{24}$$

In the second step, we analyse the quantization error and the dither signal.

3.2.1 High Bit-Rate Quantizer

At high bit-rates, the quantization process can be approximated by an additive and uncorrelated white noise. This means the cross-correlation term R_{qd} in Eq. (24) is zero; therefore, Eq. (24) is simplified to

$$\bar{\sigma}_e^2 = \frac{1}{M} Trace[s^T R_{qq} s] + \frac{1}{M} Trace[s^t R_{dd} s] \tag{25}$$

At high bit-rates, we can consider the quantization noise as a memoryless signal; therefore, R_{qq} is a diagonal matrix. The elements on the main diagonal are the variances of the quantization noise of subbands and can be calculated using Eq. (2)

$$R_{qq}(i,i) = E[e_{q_i}^2] = \frac{\Delta_i^2}{12} + \sum_{b=1}^{+\infty} \frac{(-1)^b}{(\pi b/\Delta_i)^2} M_{x_i}(\frac{j2\pi b}{\Delta_i}).M_{d_i}(\frac{j2\pi b}{\Delta_i}) \tag{26}$$

where M_{d_i} is the characteristic function of the dither signal in the i[th] subband.

The dither signal is usually selected to be a memoryless signal with bipolar, Gaussian or uniform distributions. In these cases, the diagonal element of R_{dd} is equal to the dither variance in each channel.

3.2.2 High-Bit Rate Quantizer + Paraunitary Filter Bank

For the of the paraunitary filter bank, the subband signals are orthogonal and we have

$$\sum_{i=0}^{L-1} Q_i Q_{i+j} = \begin{cases} I & j = 0 \\ 0 & j \neq 0 \end{cases} \tag{27}$$

We can use Eq.(27) and expand the two terms in Eq. (25).

$$\bar{\sigma}_e^2 = \frac{1}{M} \sum_{i=1}^{M} \{\sigma_{q_i}^2 + \sigma_{d_i}^2\} \sum_{j=1}^{N} g_i^2(j) \tag{28}$$

3.2.3 Low Bit-Rate Quantizer + Perfect Reconstruction Filter Bank

When we apply quantization watermarking to the natural signal, the quality of reconstructed signal should be high; therefore we should use high bit-rate quantizer for the lowest frequency subband. However we can have high quality image, even with coarse quantization of high frequency subbands which usually have a memoryless Laplacian signal [6].

If we assume Laplacian distribution for high frequency subbands, and high bit-rate quantization for lowest frequency subband, we can approximate all the non-diagonal elements of quantization matrix R_{qq} with zero. The diagonal elements of R_{qq} are the energies of quantization noise in subbands ($R_{qq}(i, i) = \sigma_{q_i}^2$).

We can derive a closed formula for the quantization error for high frequency subbands having Laplacian distributions. For the normalized Laplacian source, the characteristic function is

$$M_x(ju) = \frac{1}{1 + 0.5u^2} \qquad (29)$$

Therefore, Eq. (3) for high frequency subbands can be written as

$$R_{qq}(i,i) = \bar{\sigma}_{q_i}^2 = \frac{\Delta_i^2}{12} + \Delta_i^2 \sum_{b=1}^{+\infty} \frac{(-1)^b}{\pi^2 b^2} \frac{1}{1+2(\frac{\sigma_{x_i}\pi b}{\Delta_i})^2} M_{d_i}(j2\pi b \frac{\sigma_{d_i}}{\Delta_i}) \qquad (30)$$

and Eq. (5) can be simplified to

$$E[e_i d_i] = \sum_{b=1}^{+\infty} \frac{\Delta_i^2 (-1)^{b+1}}{\pi b} \frac{1}{1 + 2(\frac{\sigma_{x_i}\pi b}{\Delta_i})^2} \mathrm{Im}\{M_d^1(j2\pi b \frac{\sigma_{d_i}}{\Delta_i})\} \qquad (31)$$

4 EXPERIMENTAL RESULTS

In order to verify the analytical results, we need a large number of experiments on different configurations of filter banks, bit-allocations and different input data-sets and dithering. In this paper, we only provide some results on testing Eq. (31), which is a key factor in watermark detection for the "Lenna" image using bipolar dithering. For the bipolar dither, we have

$$d[n] = \pm\sigma_d \qquad (32)$$

$$M_d(ju) = \cos(u) \qquad (33)$$

$$M_d^1(ju) = j\sin(u) \qquad (34)$$

We implemented a single level wavelet decomposition using Daubechies 12-tap filters. The value of $\left|E[e_i d_i]/\sigma_{d_i}^2\right|$ is calculated based on Eq. (31). As shown in Fig. 4, we have accurate match with the analytical results, especially in the in the mid-range of (Δ).

5 CONCLUSION

In this paper, we derived a general formulation for the reconstruction error and watermark detection in quantization watermarking in the filter bank domain. We did not assume any constraint on the type of filter bank and quantizers. We also simplified the mathematical formulation for the high and low bit rates. Some experimental results for embedding data in an image subband coder are reported.

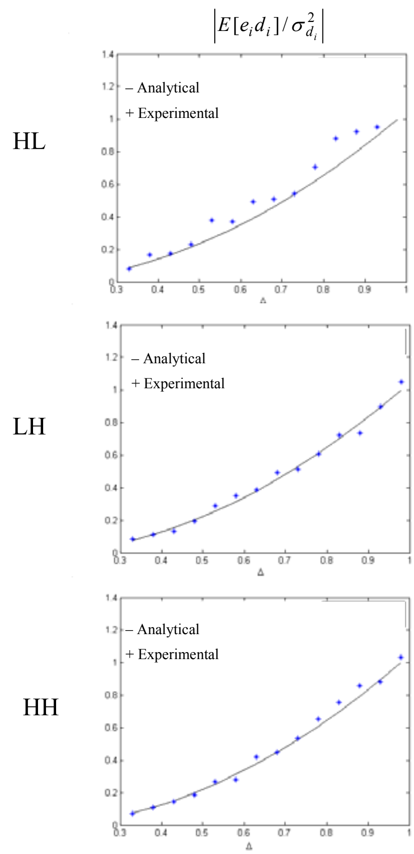

Fig. 4. Cross-correlation between the quantization noise and added watermark

REFERENCES

[1] J. Eggers and B. Girod. "Quantization effects on digital watermarks". *Signal Processing*, Vol.81, No.2, pp. 239–263, 2001.

[2] R. M. Gray and T. G. Stockham. "Dithered quantizers". *IEEE Trans. on Information Theory*, Vol. 39, No.5, pp. 805–812, 1993.

[3] K. Nayebi , T.P.I. Barnwell and M.J.T. Smith. "Time - domain filter bank analysis: A new design theory". *IEEE Transaction. on Signal Processing*, Vol.44, No. 6, pp. 1412–1429, 1992.

[4] I. Sodagar, T.P.I Barnwell and M.J.T. Smith. "On the statistically optimum FIR filter bank design". *IEEE Int. Conf. on Acoustic, Speech and Signal Proceeding,* pp. 227–230, 1994.

[5] M. Ashourian, Z. Mohdyusof. "Scalar quantization error analysis for image subband coding using dithering". SPIE Int. Conf. on Visual Comm. and Image Proc., Vol. 3, pp. 1434–1442, Australia 2000.

[6] K.A. Birney, and T.R. Fischer. "On the modeling of DCT and subband image data for compression". *IEEE Transactions on Image Processing*, Vol.4, No.2, pp. 186–193, 1995.