Spectral Coding of Three-Dimensional Mesh Geometry Information Using Dual Graph

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Abstract. In this paper, we propose a new scheme for the geometry coding of three-dimensional (3-D) mesh models using a dual graph. In order to compress the mesh geometry information, we generate a fixed spectral basis using the dual graph derived from the mesh topology. After we partition a 3-D mesh model into several independent submeshes to reduce coding complexity, each submesh geometry is projected onto the generated orthonormal basis for the spectral coding. We encode two initial vertices and the dual graph information of the mesh geometry and prove the reversibility between the dual graph and the mesh geometry. The proposed scheme overcomes difficulty of generating a fixed spectral basis, and it provides multi-resolution representation of 3-D mesh models.

Keywords: Spectral coding, fixed spectral basis, dual graph

1 Introduction

As the demand for high quality visual services has increased from consumers and the interest of three-dimensional (3-D) meshes has grown rapidly, it is essential to develop efficient 3-D mesh data coding methods. In general, a mesh model is simply a set of planar polygons in the 3-D Euclidean space. In order to represent a mesh surface, we can assume that the 3-D model consists of triangular faces. Basically, there are three types of information to describe the mesh surfaces: geometry, connectivity, and photometry information. In this paper, we focus on coding of the mesh geometry information.

Mesh geometry coding methods can largely be divided into two categories: spatial prediction methods and spectral methods. Deering [1] and Taubin *et al.* [2] traversed all the vertices according to the connectivity information, and then coded the vertices using a simple linear predictor in the spatial domain. Similarly, the mesh coding scheme by Touma and Gotsman [3] encoded the topology information as a traversal of the vertices in the spatial domain, and the vertex coordinates were coded by predicting them along the traversal order using a parallelogram scheme. Finally, the prediction errors were entropy-coded. On the other hand, Karni and Gotsman [4] proposed the spectral method for the 3-D mesh geometry coding. Karni and Gotsman projected the mesh geometry

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onto the basis vectors that are the eigenvectors of the mesh Laplacian matrix. Although Karni and Gotsman obtained good results with the spectral method, there are some critical problems, such as difficulty of fixed spectral basis generation and tremendous coding complexity. In this paper, we are concerned about the mesh geometry coding using the spectral method and try to solve the problem of fixed spectral basis generation.

This paper is organized as follows. Section 2 explains previous works, and Section 3 describes the proposed spectral coding method of the mesh geometry. After we provide experimental results in Section 4, we conclude in Section 5.

2 Spectral Coding of Mesh Geometry

Karni and Gotsman showed us how to extend the classical Fourier analysis to 3-D mesh data [4]. We consider a simple 3-D mesh model that is composed of n vertices. The adjacency matrix of the model is represented by the circular n x n matrix, and the diagonal matrix is represented by the n x n matrix. Finally, we can obtain the so-called Laplacian matrix from the adjacency matrix and the diagonal matrix. The Laplacian matrix describes the analog of the second spatial derivative conceptually.

The Fourier basis functions for 2-D signals are obtained as the eigenvectors of the Laplacian matrix of the graph with the topology of a 2-D grid. Karni and Gotsman extended the 2-D spectral transformation to the 3-D mesh topologies, and they performed the spectral coding by projecting the geometry data onto the eigenvectors of the Laplacian matrix. We should note that the eigenvectors are not fixed since valences of the vertices can be different. In other words, the spectral bases are different according to the mesh topology. Karni and Gotsman tried to solve the problem by mapping an arbitrary mesh topology into a regular mesh topology [5]. However, serious deformation has occurred when the arbitrary mesh is mapped into a regular mesh, as we can see in Fig. 1. In this paper, we propose a new algorithm to generate the fixed spectral basis.

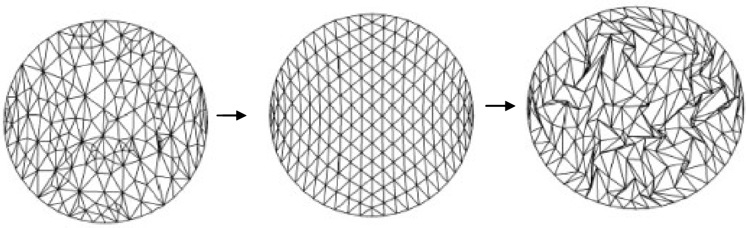

Fig. 1. Deformation problem

3 Spectral Coding Using Dual Graph

3.1 Spectral Coding Method

In order to compress the geometry information of 3-D mesh models, we propose a spectral coding using a fixed spectral basis. In the proposed scheme, we generate a fixed spectral basis from the dual graph derived from the mesh topology. In essence, we try to extend the popular 2-D transform coding approach based on the fixed spectral basis into 3-D mesh coding.

Fig. 2 shows the entire block diagram for the proposed scheme. After we analyze the input mesh model into the three types of information, we partition the mesh into several submeshes to reduce coding complexity. During the mesh partitioning operation, we obtain a dual graph from the connectivity information for each submesh. Then, we find a fixed spectral basis from the property of the dual graph. Finally, the mesh geometry is projected onto the generated fixed spectral basis, and the transformed coefficients are coded by a quantizer and a variable length coder.

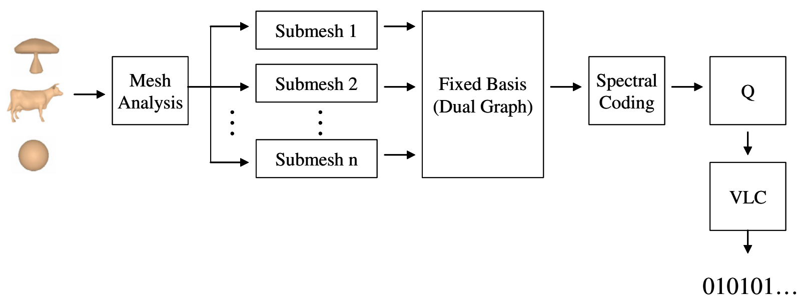

Fig. 2. Block diagram of mesh geometry spectral coding

3.2 Mesh Partitioning

Mesh partitioning is used to divide a 3-D mesh model into several independent pieces, called as submeshes. When submeshes are transmitted separately, instead of the whole 3-D mesh model, we can send the 3-D model more robustly to the receiver side since transmission errors may affect only some submeshes, not the entire model. However, the main reason that we perform the mesh partitioning in this paper is to decrease coding complexity of the spectral coding.

We apply a multi-seed traversal algorithm that is a well-known partitioning technique [6]. In the mesh partitioning, we should carefully select the initial vertices that are the starting points of the partitioning process. In this paper, we adopt the k-means clustering algorithm for initial vertex selection. When we partition a given 3-D mesh model into n submeshes, we select n vertices arbitrarily as the initial center vertices of the submeshes. Then, the initial center vertices are convergent to new center vertices by the k-means clustering algorithm. Finally, the convergent center vertices are selected as the initial vertices. Since the selected vertices are optimal positions, the mesh partitioning from these initial vertices can be performed in an optimal manner.

On the contrary, the k-means clustering algorithm needs more computation time than other algorithms, such as the maximum-distance algorithm. However, we do not require a real-time partitioning technique since the submeshes will be stored in a data storage through an off-line process. Fig. 3(a) and Fig. 3(b) show selections of the initial vertices by the maximum-distance algorithm and the K-means clustering algorithm, respectively, when the number of submeshes is three. As shown in Fig. 3, we can notice that the initial vertex positions, selected by the k-means clustering algorithm, are more optimal than those selected by the maximum-distance algorithm.

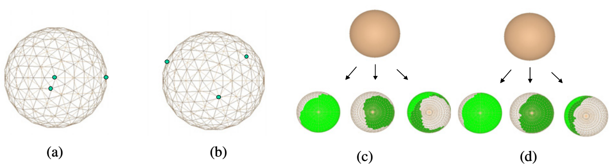

Fig. 3. Initial vertex selection (a) maximum-distance algorithm, (b) k-means algorithm, (c) the result of (a), (d) the result of (b)

3.3 Generation of the Fixed Basis Using the Dual Graph

Given a planar graph G, its geometric dual graph G* is constructed by placing a vertex in each region of G. The dual graph G* of a polyhedral graph G has dual graph vertices, each of which corresponds to a face of G and each of whose faces corresponds to a graph vertex of G [7]. Fig. 4 shows an example of a dual graph for the simple planar graph.

In this paper, we employ the dual graph to generate a fixed spectral basis. After obtaining submeshes from mesh partitioning, we generate their dual graphes by connecting the centers of gravity of triangle faces for each submesh such that we should be able to regenerate the mesh geometry from its dual graph.

As shown in Fig. 5, we should know the two initial vertices of the original mesh in order to regenerate the mesh geometry. We can construct a triangle face from two vertices, v_1 and v_2, of the original mesh and the one vertex d_1 of the dual graph. The third vertex v_3 of a triangle face is located on the defined plane. When we calculate the midpoint t_1 from v_1 and v_2, we can notice that the third vertex v_3 is located on the line from d_1 and t_1. Finally, we can find the third vertex v_3 by advancing two times of the distance between d_1 and t_1 from d_1. With this procedure, we regenerate the mesh geometry properly. In other

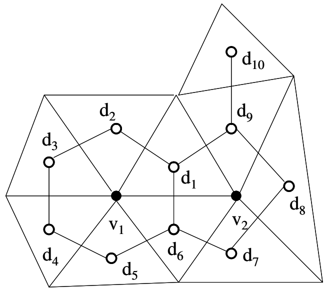

Fig. 4. Polyhedral graph and its dual graph

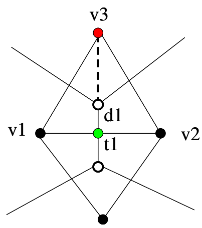

Fig. 5. Regeneration of mesh geometry with its dual graph

words, we can replace the mesh geometry with the first two initial vertices of the original mesh geometry and its dual graph information. Therefore, we encode the dual graph information, instead of the original mesh geometry.

The main problem of generating a fixed spectral basis is the irregularity of the mesh. We solve the problem by using the duality between the mesh topology and its dual graph. Since the 3-D mesh model is composed of triangular faces, valences of most vertices in a dual graph vertices are 3. This property is the key idea to generate a fixed spectral basis.

As shown in Fig. 4, the valence of each vertex in the dual graph is 3 when the triangle face is the inner face in the 3-D mesh model. When the triangle face is located on the boundary of the model and is not an ear, the valence of each vertex in the dual graph is 2. In case the triangle face is an ear, we can assume that the valence of each vertex in the dual graph is 1. With the assumption, we can generate the fixed spectral basis by analyzing each submesh topology.

In general, we can extract the boundary information at the decoder side since the topology information is transmitted prior to the mesh geometry information. As a result, we can generate the spectral basis with the extracted boundary information and the number of inner triangles.

3.4 Spectral Coding of Mesh Geometry

After the dual graph vertices are projected onto the generated basis, the spectral coefficients are coded by a quantizer and a variable length coder. In general, the spectral coefficients are uniformly quantized using between 10 bits and 16 bits. Finally, the quantized coefficients are entropy coded using a Huffman or arithmetic coder. We can provide the progressive transmission and multi-resolution representation of 3-D meshes by selecting the spectral coefficients to be sent.

4 Experimental Results and Analysis

We have evaluated the proposed algorithm with the COW model. It consists of 2903 vertices and 5804 faces. We partitioned the COW model into 20 submeshes. We used the Hausdroff distance [8], which is the common measure of 3-D model deformation, to compare performances of the proposed algorithm and other algorithms.

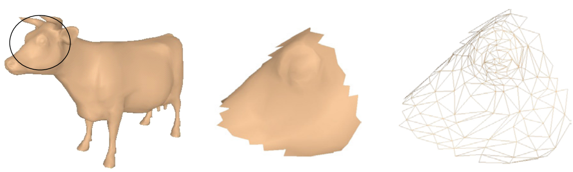

Fig. 6. 19^{th} submesh of the COW model

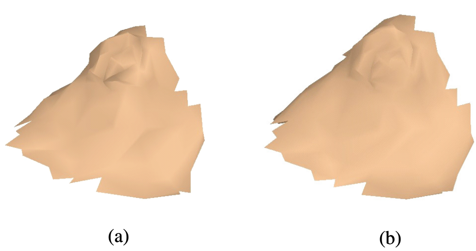

Fig. 7. Result of the 19^{th} submesh (a) basis generation using a regular mesh, (b) basis generation using a dual graph

Fig. 6 shows the 19^{th} submesh of the COW model, and Fig. 7 shows the result of spectral coding. We generated 196 fixed spectral bases for the 19^{th} submesh. In our experiment, we coded 150 spectral coefficients of 196 spectral coefficients.As shown in Fig. 7, we had some deformations in both algorithms. However, our

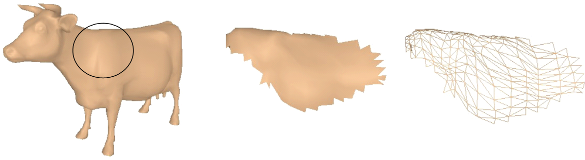

Fig. 8. 4^{th} submesh of the COW model

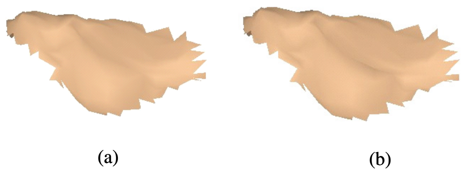

Fig. 9. Result of the 4^{th} submesh (a) basis generation using a regular mesh, (b) basis generation using a dual graph

Table 1. Comparison of Hausdroff distances

Submesh	Vertices	Coded Coefficients	Regular mesh	Proposed Scheme
1^{st}	182	130	0.0055	0.0059
2^{nd}	156	130	0.0079	0.0062
3^{rd}	158	130	0.0069	0.0059
4^{th}	186	150	0.0104	0.0118
5^{th}	187	150	0.0072	0.0068
6^{th}	182	150	0.0058	0.0051
7^{th}	152	130	0.0046	0.0049
8^{th}	177	130	0.0102	0.0093
9^{th}	174	150	0.0046	0.0039
10^{th}	159	130	0.0064	0.0048
11^{th}	185	150	0.0088	0.0075
12^{th}	159	130	0.0041	0.0036
13^{th}	172	150	0.0047	0.0039
14^{th}	198	150	0.0072	0.0071
15^{th}	188	150	0.0042	0.0049
16^{th}	164	130	0.0051	0.0045
17^{th}	161	130	0.0027	0.0031
18^{th}	174	130	0.0045	0.0041
19^{th}	170	150	0.0058	0.0035
20^{th}	173	150	0.0069	0.0052

proposed algorithm had less deformation than the previous algorithm in the left eye region of the COW model, because there was mismatching between the original mesh and a regular mesh in the previous algorithm.

Fig. 8 shows the 4^{th} submesh of the COW model, and Fig. 9 shows the results of the 4^{th} submesh. As we can see in Fig. 8, valances of the 4th submesh vertices are all 6. As a result, the previous algorithm worked better than ours. Our algorithm had an error propagation problem when we regenerated the mesh geometry from the dual graph. We can overcome the problem by inserting a original mesh geometry information at the encoder side randomly to refresh the propagated errors. Table 1 shows the Hausdroff distance for other submeshes.

5 Conclusions

In this paper, we proposed a new algorithm for 3- D mesh geometry coding. We generated the fixed spectral basis from the dual graph derived from the mesh topology. After we generated the fixed spectral basis, we projected the mesh geometry onto the basis. Then, we coded the spectral coefficients using a quantizer and a variable length coder. The proposed algorithm can reduce the deformation after spectral coding and minimize coding complexity using mesh partitioning. The proposed algorithm can be used for progressive transmission and multi-representation of 3-D mesh models.

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References

1. Deering, M.: Geometry Compression. Proceedings of SIGGRAPH (1995) 13-20
2. Taubin, G., Rossignac, J.: Geometric Compression through Topological Surgery. ACM Transactions on Graphics (1998) Vol. 17 84-115
3. Touma, C., Gotsman, C.: Triangle Mesh Compression. Proceedings of Graphics Interface (1998) 26-34
4. Karni, Z., Gotsman, C.: Spectral Compression of Mesh Geometry. Proceedings of SIGGRAPH (2000) 279-286
5. Karni, Z., Gotsman, C.: 3D Mesh Compression using Fixed Spectral Basis. Proceedings of Graphics Interface (2001) 1-8
6. Yan, Z., Kumar, S., Li, J., Kuo, C-C.J.: Robust Coding of 3D Graphics Models using Mesh Segmentation and Data Partitioning. Proceedings of IEEE International Conference on Image Processing (1999) 25-28
7. Joseph O.:Computational Geometry in C, Cambridge University Press (1998)
8. Chang, E.Y., Ho, Y.S.: Three-dimensional Mesh Simplification by Subdivided Edge Classification. Proceedings of IEEE Region 10 Annual Conference (2001) 1-4