Reasoning about Models of Context

A Context-Oriented Logical Language for Knowledge-Based Context-Aware Applications

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ABSTRACT. *Ontologies have been suggested as a means to ensure interoperability of Ambient Intelligence (AmI) applications as well as effectiveness within different sensory environments. We introduce a new logical language specifically tailored to the task of ontology design for AmI applications. This language has known advantages of description logics, namely a precisely defined semantics and a brief, intuitive syntax; and it facilitates formalising knowledge about contexts: a unified syntax for representing containment relations makes it easy to specify that a context is spatially or temporally contained in another, that two contexts share a certain group of agents or objects, or that an application is in a state that requires it to adapt its behaviour.*

RÉSUMÉ. *Les ontologies ont été proposées comme un moyen d'assurer l'interopérabilité des applications de l'Intelligence Diffuse (AmI en anglais) ainsi que leur efficacité au sein de différents environnements sensoriels. Nous proposons un nouveau langage logique dédié à la tâche de conception d'ontologies pour des applications de l'Intelligence diffuse. Ce langage possède les avantages connus des logiques de description, à savoir une sémantique définie précise et une syntaxe simple et intuitive. Il facilite la formalisation des connaissances sur les contextes. Une syntaxe unifiée pour représenter les relations d'endiguement facilite la spécification d'un contexte spatialement ou temporellement inclus dans un autre contexte, et que les deux contextes partagent un certain groupe d'agents ou d'objets, ou qu'une application est dans un état qui lui demande d'adopter tel comportement.*

KEYWORDS: *Contextual reasoning, Representation of context, Logical language, Ontologies, Ambient Intelligence, Mereotopology*

MOTS-CLÉS : *Raisonnement contextuel, Représentation du contexte, Langage logique, Ontologies, Intelligence diffuse, Méréotopologie*

1. Introduction

A fundamental question for any research on context-aware, intelligent computer systems is how to represent context. As perspectives of research and targeted types of context-awareness differ, the specific properties of representations of context also differ; cf. the range of perspectives surveyed by Brézillon (1999). For many context-aware applications, we need to integrate several of these perspectives and find interfaces and mappings between different types of representations of context. A useful assumption for this endeavour is that there is a core concept of *context* shared under different perspectives. In this article, we present a logic-based approach to the representation of context, which was developed, on the one hand, as a formal foundation for a broad range of context-aware applications including Ubiquitous Computing and Virtual Heritage Systems and, on the other hand, as a knowledge representation framework for contextual reasoning.

When human beings reason or communicate about objects and events in the environment, they usually abstract from certain aspects, and reason within a context. When we reason about space, for instance, we may use *to the West of* as a transitive relation; cf. the cardinal direction calculus of Ligozat (1998). This assumption is valid as long as we suppose a sufficiently small local area of context, as *to the West of* is globally a cyclic relation: Denmark is to the West of Korea, Korea is to the West of Canada, and Canada is to the West of Denmark; within the local context of a city or country, in contrast, *to the West of* can be used in the same manner as *to the North of*, i.e. as if it was a transitive, acyclic relation. Research in *contextual reasoning* investigates how such locally valid theories can be connected and how inferences can be made across context boundaries. Benerecetti *et al.* (2000) study the basic principles and tasks of contextual reasoning. They suggest the metaphor of *context as a box* containing the contextualised logical sentences together with a list of parameters and values for these. These parameters establish the link between the contextualised sentences and the surrounding or neighbouring boxes. Three dimensions of context dependence are distinguished (Benerecetti *et al.*, 2000): contextualised representations can be *partial*, *approximate*, or *perspectival* representations. The three dimensions of context dependence can be illustrated with respect to the parameter of spatial context: changing from a global to a partial view of space can be identified with spatially focusing on a sufficiently small local area; a change of spatial perspective can be identified with a change of reference frames; and changing the degree of approximation can be related to coarsening and refinement of spatial granularity. The focus of this article is on the aspect of *partial* representations.

Research on context-aware applications in ubiquitous computing has concentrated on the question how to obtain and process information about the context of a user or application. The notion of contextual information has been characterised by Dey *et al.* (2000) as "any information that can be used to characterise the situation of an entity." With an entity being defined as "a person, place, or object that is considered relevant to the interaction between a user and an application, including the user and applications themselves." Mobile context-aware computing has to cover issues of sensor reliability, ad hoc network communication, software development support, reasoning and inference, usability, and privacy management. Many of the main differences between approaches can accordingly be traced back to emphasis on one or the other aspect, such as sensor-fusion (Schmidt *et al.*, 1999), networking (Schilit *et al.*, 1994), and development of context-aware applications (Henricksen *et al.*, 2006). Most recently, ontology-based approaches (Strang *et al.*, 2003; Ranganathan *et al.*, 2003b; Gu *et al.*, 2005) have gained importance to answer the demands of heterogeneous application environments.

However, Ye *et al.* (2007) point out that ontologies for pervasive computing systems are particularly hard to develop, as they have to contain concepts for dealing with the physical world as perceived through sensors and with the common sense notions of spatial and temporal *context* a user or developer has. Both problems are not well supported by standard ontology languages (Sect. 2).

We propose a new logical language, for brevity called *context logic*, that aims to unite the contextual reasoning perspective with the pervasive computing perspective. The logical language was designed to provide a mathematically as well as physically and computationally meaningful semantics for contextual information. On the computational side, it functions as a logical foundation and knowledge-based reasoning system for an application model for context-aware applications (Jang *et al.*, 2005; Jang *et al.*, 2003, sketched in Sect. 3): the language can describe classes of situations as well as classes of actions to be triggered by situations. With respect to the physical aspect, we outline how input from sensors together with metadata on the accuracy of the sensor can be translated into logical formulae that reflect uncertainty (Sect. 4). On the mathematical side, we give a formal definition of the syntax and semantics (Sect. 5) using standard techniques from modal logic, which make it easy to compare the language with other ontology languages, such as description logics. We prove soundness and weak completeness (Sect. 6). We summarise results, open questions and future work in Sect. 7.

2. Related Works

Ontologies support three major issues in the development of context-aware ubiquitous computing applications (Ranganathan *et al.*, 2003b): discovery and matchmaking, interoperability, and context-awareness. Ontology languages based on description logics (Nardi *et al.*, 2002), such as the ontology web language (OWL), best support formulation of taxonomic knowledge. Representing knowledge about the classical domains of context, in particular space and time however, is not well supported in OWL (Ye *et al.*, 2007). Several approaches to ontology-based context modelling employ additionally more expressive logical languages (Ranganathan *et al.*, 2003b; Gu *et al.*, 2005; Strang *et al.*, 2003). Ranganathan *et al.* (2003b) believe that this need is caused by space and time being quantitative domains. However, research in qualitative reasoning has shown that space and time can be reasoned about efficiently with qualitative representations (Renz, 2007), and that users are more comfortable with qualitative than with quantitative interfaces to spatial knowledge (Egenhofer, 1994). Qualitative knowledge about specific domains can be handled in a tractable way by using specialised logical languages, such as spatial logics (Bennett, 1994), or description logics for taxonomic reasoning (Donini, 2003). However, combining logics of low complexity can result in logics of high complexity (Gabbay *et al.*, 2003). Combinations of logical languages that retain low complexity have been explored by Kutz *et al.* (2004).

To see that it is not trivial to add spatial relations into taxonomic knowledge consider the integration of the RCC-relations (Randell *et al.*, 1992) in SOUPA (Chen *et al.*, 2004): the formal specification[1] expresses, for instance, only that *proper part* and *part* are transitive relations, but not that *part* is a reflexive relation and *proper part* is irreflexive; or that any proper part of a region is also a part of that region. Moreover, the SOUPA specification of geographical space[2] adds a relation *spatiallySubsumes*, which is supposed to provide spatial containment reasoning (Chen *et al.*, 2004, Sect. 3.1.5). However, a connection to the RCC-relations, or whether it is reflexive or irreflexive is not specified. The resulting compound ontology, although formally consistent, therefore does not support inferences involving combinations of RCC-regions and geographical regions.

Following a similar approach as suggested by Kutz *et al.* (2004), we characterised a specialised logical language that combines reasoning about specific domains. Our work also draws on results from the area of ontology in information systems concerning the universal applicability of the theory of mereology. An example are the mereologic first order theories for partonomic, spatio-temporal, and taxonomic structures discussed by Bittner *et al.* (2004). At the centre for each mereologic domain theory is a *part-of* or *containment* relation, a partial ordering relation that spans a semi-lattice structure, representing the *containment hierarchy*, over some domain.

The goal of our research was to support a context model for context-aware applications (Jang *et al.*, 2005) with a logical semantics and a contextual reasoning mechanism. Jang *et al.* (2005) suggested that the context of some focus entity, such as an application, device, smart space, virtual agent, user, or task, can be described appropriately if we know *who* interacts *how* and for which task (*why*) with *what when* and *where*. Jang *et al.* (2005) accordingly called their context model the *5W1H context model*. At the basis of the proposed log-

1. `http://pervasive.semanticweb.org/ont/2004/06/rcc`
2. `http://pervasive.semanticweb.org/ont/2004/06/space`

ical language is the assumption that a partial ordering structure – and thus a containment or implication hierarchy – for handling these six aspects of context is a minimum requirement for context-aware applications. A spatial containment hierarchy, for instance, is a core component for location-aware systems (Leonhardt, 1998).

3. A Framework for Developing Context-Aware Applications

Before we describe the proposed logical formalism, we shortly discuss how context-adaptive behaviour is generated using a knowledge-based reasoning system for context logics installed on a ubiquitous computing infrastructure, and we motivate requirements on the logical language with respect to examples from related works on ontology-based context-adaptive systems. The knowledge-based reasoning system is designed to become a part of an intelligent assistant system (Brézillon *et al.*, 1999) that can help users to control and interact with context-aware computing systems. The basic infrastructure is provided by the Unified Context-Aware Application Model (UCAM). UCAM supports interoperability between devices and applications in smart spaces (Jang *et al.*, 2003), multimedia contents in virtual and augmented reality (Lee *et al.*, 2004), and wearable and other personal devices (Hong *et al.*, 2006); it encapsulates the network connections that link sensors and services in a middleware framework; and it provides the generic data format for collecting, transmitting, and storing contextual data, the 5W1H context model suggested by Jang *et al.* (2005) and further developed by Hong *et al.* (2007).[3]

In contrast to conventional desktop applications, context-aware applications cannot be developed in isolation from the contexts in which they are going to be used (Vieira *et al.*, 2007). We assume that at least three types of actors contribute to designing and configuring a context-adaptive system: (1) developers of core applications provide devices and standard applications offering standard computational functionalities; (2) local domain knowledge engineers or administrators – including intranet or groupware providers, smart space providers, but also VR designers – link the application into a specific domain; (3) users link their personal devices with certain spaces and applications and, using personalisation mechanisms, configure applications as they desire (Fig. 1). Interoperability plays a major role in this model, since the actual behaviour of an application in a context depends not only on the application programmers together with a set of parameters as in conventional computing, but indeed there is no running application before administrators and users add their particular components. The burden of knowledge acquisition (Brézillon *et al.*, 1999)

3. A discussion of the functionality of UCAM is beyond the scope of this article. A complete application example demonstrating sensor fusion for context integration is given by Oh *et al.* (2004); basic data structures have been discussed by Hong *et al.* (2007).

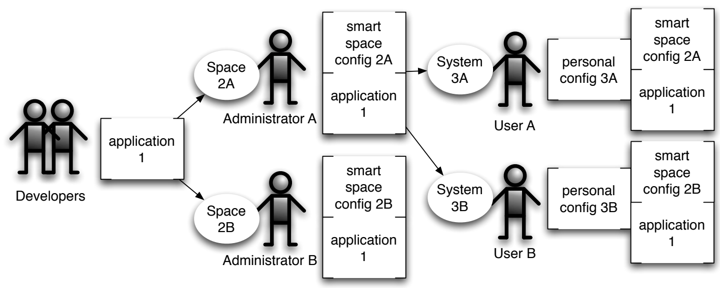

Figure 1. *By definition, context-aware application behaviour has to be highly context-dependent. Context-aware applications depend to a larger degree on specific domain knowledge with respect to environments and users*

is in our approach distributed between developers, administrators, and users. The result is that the context-dependent behaviour varies with users, tasks, environments, groups, and applications. We assume like Vieira *et al.* (2005) that the link between the different parts is mainly established by ontologies, which allow for interoperability and information sharing. However, we argue that a specialised logic framework is needed to develop ontologies supporting context-aware classification and adaptation mechanisms.

It has been widely recognised (Ye *et al.*, 2007) that description logics (Nardi *et al.*, 2002) – which through OWL are becoming the de facto standard for ontology specification (Shadbolt *et al.*, 2006) – are not equally suitable for modelling all aspects of context. And although a number of ontology-based approaches to context modelling use OWL for specifying taxonomic knowledge, all of them additionally use external tools (Heckmann, 2005) or a more powerful logical language, such as first order logic (Gu *et al.*, 2005; Strang *et al.*, 2003; Heckmann, 2005; Vieira *et al.*, 2005; Ranganathan *et al.*, 2003b) or F-Logic (Strang *et al.*, 2003) to formally specify central aspects of context, namely spatial and temporal containment, basic quantitative comparison mechanisms, and task structures (Brézillon, 2005) or triggering of rules. In contrast, approaches that integrate all these aspects of context into a purely OWL-based ontology framework (Chen *et al.*, 2004) result in ontologies that provide rich taxonomies about relationships between categories of objects or persons – the *who* and *what* of context –, but only sparse support for the other aspects of context.

Although OWL is certainly expressive enough to represent all six aspects of context, it is clear that it enforces an object-oriented perspective: the basic distinction is that of instances (objects) and concepts (classes). This perspective seems artificial for context-aware computing. Our starting point for the definition of a more suitable context-oriented logical language, and the basic assumption underlying the 5W1H context model, was that all six aspects of context should be represented using a similar syntax. Additionally, the context logic framework was to have the same advantages over classical first order logic as description logics: context logic should be decidable, provide in-built support for reification over the six domains, and have a simple syntax. The most striking syntactic difference between ontologies in context logic and ontologies in first order logic or description logic is that context logic, like propositional logic, does not differentiate between instances and classes. The objective was to develop a context-oriented, mereologic perspective on context,[4] in contrast to the object-oriented, set-theoretic perspective that is assumed when we describe contexts in terms of classes and instances of classes.

4. A Context-Oriented Logical Language

From the context-oriented perspective, everything is a context, or more precisely: every portion of the world is a context. This includes, for instance: the portion of the world in which an agent Bob is present, a context that is restricted only with respect to the *who*-domain and unrestricted with respect to the other five domains; or the portion of the world that has the spatial extension of room A58, a context that is restricted only with respect to the *where*-domain. Also, we introduce two special contexts: the relevant part of the world with respect to some focus is called the *maximal context* – in the context logic formalism, named with the context term symbol $\top$ – and the *minimal context* $\bot$, which represents irrelevant, unknown, or non-existent portions of the world.

Context terms, like concepts in description logics can be combined using term operators for building the sum $\sqcup$ and the intersection $\sqcap$. Two context terms that are related with one of six *sub-context* operators form an atomic formula. Each sub-context relation refers to a containment hierarchy (Tab. 1). The relation $\sqsubseteq_{\text{what}}$ corresponds to the taxonomic relation $\sqsubseteq$ known from de-

Formula	Read as	Containment hierarchy
$[c \sqsubseteq_{\text{who}} d]$	social sub-context	groups of agents
$[c \sqsubseteq_{\text{what}} d]$	conceptual sub-context	sets of objects
$[c \sqsubseteq_{\text{when}} d]$	temporal sub-context	temporal intervals
$[c \sqsubseteq_{\text{where}} d]$	spatial sub-context	spatial regions
$[c \sqsubseteq_{\text{how}} d]$	conditional sub-context	implied conditions and states
$[c \sqsubseteq_{\text{why}} d]$	task sub-context	tasks

Table 1. *Sub-context operators with corresponding containment hierarchies*

scription logics. In addition, context logic allows for five other types of relations.

4. Cf. Bittner *et al.* (2004) and Eschenbach (2004) as examples of recent approaches discussing mereology with respect to general ontologies.

Spatial subsumption, for instance, is represented by the operator $\sqsubseteq_{\text{where}}$ (more examples are given below). *Roles* and *quantification* known in description logics have no counterparts. But, unlike description logic, context logic allows for propositional connectives to combine formulae. We can thus describe complex relationships between contexts, such as $[c \sqsubseteq_{\text{when}} d] \to [c \sqsubseteq_{\text{where}} d]$, stating "if c is a temporal sub-context of d then c is also a spatial sub-context of d." This statement would, for instance, make sense for describing the space-time portions occupied by a moving object.

In contrast to description logic and first order logic, context logic provides no special syntactic means to refer to individuals. The context logic expression $[\text{group1} \sqsubseteq_{\text{who}} \text{deptA}]$ – saying that all agents in group 1 also belong to department A – could be stated in description logics as $\text{group1} \sqcap \text{who} \sqsubseteq \text{deptA}$, meaning that all objects of group1 that are in the who-class are also objects of the class deptA. Similarly, we would state in first order logic: $\forall x : \text{group1}(x) \wedge \text{who}(x) \to \text{deptA}(x)$. With a sorted first order logic, used for instance by Ranganathan *et al.* (2003a), we could declare *who* to be a sortal predicate and state $\forall_{\text{who}} x : \text{group1}(x) \to \text{deptA}(x)$. For the statement $[\text{alice} \sqsubseteq_{\text{who}} \text{group1}]$, we can either go the same way, translating it into the DL-expression $\text{alice} \sqcap \text{who} \sqsubseteq \text{group1}$, so that Alice is represented as the concept describing only Alice, or we can state that Alice is an instance of the group 1: group1(alice). In first order logic, this distinction would be reflected by the formalisations $\forall x : \text{alice}(x) \wedge \text{who}(x) \to \text{group1}(x)$ and $\text{group1}(\mathit{alice})$, respectively. With the latter alternative, we lose the ability to refer to context aspects other than the who-aspect of Alice.

Before we introduce the formal definition of the context logic in the next section, we illustrate the basic requirements on the logical language with respect to a prototypical example: a context-aware jukebox as specified by Ranganathan *et al.* (2003a). For simplicity, we focus on how location models (*where*-domain) and knowledge about states (*how*-domain) and tasks (*why*-domain) can be specified.

Context logics are expressive enough to encode classes of quantitative values as received from a sensor. This comparison of quantitative values is necessary in pervasive computing for triggering actions depending on a sensory value being within a certain interval. We discuss an example of a first order logic rule in the context-aware jukebox application (Ranganathan *et al.*, 2003a, p.360):

$$\text{IF Location(Alice, In, 2401) AND Temperature(Champaign,} >, 50) \\ \text{THEN PlayHappyMusic()}$$

For assuming a context-oriented perspective, we first have to determine the focus of the context, in the example, the context-aware jukebox. The jukebox is installed in room 2401. Under the context-oriented perspective we state this directly: the relevant portion of the world ($\top$) from the perspective of the jukebox is with respect to space the room 2401, a location that is spatially subsumed by the geographic location *Champaign* [J1]. With respect to world states in the *how*-domain, the jukebox can detect only one class of temperatures $t \in (50, \infty)$ together with its complement, and it has information about the state of the stock market and the music it is currently playing [J2].

$$[\top \sqsubseteq_{\text{where}} \text{r2401}] \wedge [\text{r2401} \sqsubseteq_{\text{where}} \text{champaign}] \quad \text{[J1]}$$

$$[\top \sqsubseteq_{\text{how}} \text{temp} \sqcup \text{stock} \sqcup \text{music}] \wedge [\text{higher50F} \sqsubseteq_{\text{how}} \text{temp}] \quad \text{[J2]}$$

One question is how to model the resulting actions of the jukebox. Ranganathan *et al.* (2003a) intend *PlayHappyMusic* to be “a function that is implemented by the application and that is called by the [...] reasoning engine.” An alternative approach, which allows us to reason about alternative actions and their results, is to interpret *PlayHappyMusic* as a context in which it is a task to play happy music. We can then describe the rule with a complex context logic formula [J3]: if one of the users present is Alice and the temperature is higher than $50°F$ then one task – i.e. an action that is not contained in the minimal, irrelevant context – is to play happy music:

$$\neg[\text{alice} \sqsubseteq_{\text{who}} \bot] \wedge [\text{temp} \sqsubseteq_{\text{how}} \text{higher50F}] \rightarrow \neg[\text{playHappyMusic} \sqsubseteq_{\text{why}} \bot] \quad \text{[J3]}$$

Quantitative information from sensors about changing states of the world can be modelled in terms of intervals (Chalmers *et al.*, 2004; Schmidtke *et al.*, 2006), which also allows for modelling uncertainty. Assume the temperature sensor delivers information with an accuracy of $\pm 5F$. Then, a sensor value of $40F$ can be interpreted as a statement from the sensor that the actual value is in the interval $(35F, 45F)$. Uncertainty of sensory measurement can thus be encoded explicitly. Assume the temperature is first in the interval $(35F, 45F)$ in a context c_0, then gradually increases first to $(45F, 55F)$ at c_1, and finally reaches $(53F, 63F)$ at c_2. We obtain information from sensors by using logical sensors that understand the numerical values and deliver context logic terms to the application: the actual temperature value for c_0 is clearly outside the interval $(50, \infty)$ [V1] and it is clearly inside for c_2 [V3]; for c_1, the actual status is uncertain, as the two intervals overlap [V2].

$$[\langle 35F, 45F \rangle \sqsubseteq_{\text{how}} -\text{higher50F}] \quad \text{[V1]}$$

$$\neg[\langle 45F, 55F \rangle \sqcap \text{higher50F} \sqsubseteq_{\text{how}} \bot] \quad \text{[V2]}$$

$$[\langle 53F, 63F \rangle \sqsubseteq_{\text{how}} \text{higher50F}] \quad \text{[V3]}$$

In this way, we can model fluent values retrieved from sensors. Situation c_2 could be described as the (relevant part of the) temperature being in the interval $(53F, 63F)$, and both Alice and Bob being (relevant) agents in the context:

$$[\text{temp} \sqsubseteq_{\text{how}} \langle 53F, 63F \rangle] \wedge \neg[\text{alice} \sqsubseteq_{\text{who}} \bot] \wedge \neg[\text{bob} \sqsubseteq_{\text{who}} \bot] \quad \text{[SD]}$$

Automatic reasoning can derive that playing happy music is a relevant task in c_2, since $\neg[\text{playHappyMusic} \sqsubseteq_{\text{why}} \bot]$ is a consequence of the jukebox application ontology [J1]-[J3], the ontology of values managed by the logical sensor [V1]-[V3], together with the situation description for c_2 [SD].

5. Formal Specification of Syntax and Semantics

There are two types of expressions in description logics: concepts and formulae, where only concepts are recursively defined. Similarly, the logical language defined in this paper consists of the recursively defined context terms, referring to contexts, and formulae.

Definition 1. *The set of context terms CTerm is defined based on a set of context variables CTVar, together with two additional symbols* $\top$*, for the* maximal context*, and* $\bot$*, for the* minimal context*. The set CTerm of* context terms *is the smallest set that fulfils:*

1) The context variables $a \in CTVar$*, and the special symbols* $\top$ *and* $\bot$ *are atomic context terms.*

2) If c *and* d *are context terms, then the* complement $-c$*, the* sum $(c \sqcup d)$*, and the* intersection $(c \sqcap d)$ *are also context terms.*

We omit brackets, in particular outer brackets, if no ambiguity can arise.

Description logics provide a general mechanism of roles and quantification for specifying relations such as spatial subsumption. Instead of this, context logic supports exactly six operators for generating formulae out of context terms.

Definition 2. *The set CFor of all context logic formulae is defined recursively as the smallest set containing the following expressions:*

1) If c *and* d *are context terms, then* $[c \sqsubseteq_{\text{who}} d]$, $[c \sqsubseteq_{\text{what}} d]$, $[c \sqsubseteq_{\text{when}} d]$, $[c \sqsubseteq_{\text{where}} d]$, $[c \sqsubseteq_{\text{how}} d]$, $[c \sqsubseteq_{\text{why}} d]$ *are atomic context formulae.*

2) If ϕ *and* ψ *are formulae, then the negation* $\neg\phi$*, the disjunction* $(\phi \vee \psi)$*, and the conjunction* $(\phi \wedge \psi)$ *are also formulae.*

For a context term c *other than the special symbols* $\top$ *and* $\bot$*, the formula* $[\top \sqsubseteq_m c]$ *is called the* positive atom *of* c*, and* $[c \sqsubseteq_m \bot]$ *is called the* negative atom *of* c*. We will assume a function* $\text{varN} : CFor \rightarrow \mathbb{N}$ *giving the number of occurrences of context variables* $a \in CVar$ *in* ϕ*. We leave out the formal definition for brevity, and only show some examples for illustration:* $\text{varN}([\top \sqsubseteq_{\text{who}} \top]) = 0$, $\text{varN}([\top \sqsubseteq_{\text{who}} a]) = 1$, $\text{varN}([a \sqsubseteq_{\text{when}} a]) = 2$, $\text{varN}([a \sqsubseteq_{\text{where}} a] \wedge [b \sqsubseteq_{\text{where}} \bot]) = 3$*. A* contextual knowledge base *(CKB) is formally defined as a set of context formulae.*

We can now specify the semantics of the language. We characterise a Kripke style semantics as used in modal logics, which allows us to interpret knowledge about context in a local, context-dependent manner. In the definitions, we are following mainly the notation of the textbook of Chagrov *et al.* (1997). We define a model M as $M = \langle F, V \rangle$. Here, F is a multi-modal Kripke frame $F = \langle W, R \rangle$ consisting of a non-empty set of contexts W – the worlds of the

Kripke frame – and a six-tuple $R = \langle R_{\text{who}}, R_{\text{when}}, R_{\text{where}}, R_{\text{what}}, R_{\text{how}}, R_{\text{why}} \rangle$ of partial ordering relations for interpreting the six sub-context relations. V denotes a six-tuple $\langle V_{\text{who}}, V_{\text{when}}, V_{\text{where}}, V_{\text{what}}, V_{\text{how}}, V_{\text{why}} \rangle$ of valuation relations for interpreting atomic context terms, i.e. context variables and the special symbols $\top$ and $\bot$.

Intuitively, each context $w \in W$ can be thought of as a six-tuple of sections of the six domains. The relations R_m, with $m \in \{\text{who}, \text{when}, \text{where}, \text{what}, \text{how}, \text{why}\}$ can be understood as containment relations between the respective sections. Accordingly, we read $R_m(w, w')$ either as w *m-contains* w', referring to the intended model, or as w' is *m-accessible* from w, referring to its role in Kripke semantics. Formally, we characterise each R_m as a preorder giving rise to a partial order on the m-equivalence classes of W. We define m-equivalence as equivalence with respect to R_m [D1]. We demand that each R_m be reflexive [A1] and transitive [A2]. It follows from transitivity and the definition of E_m that R_m is antisymmetric up to m-equivalence. We demand [A3] that a context is uniquely characterised by the six aspects.

$$E_m(w_1, w_2) \overset{def}{\Leftrightarrow} \quad \forall w : [R_m(w_1, w) \leftrightarrow R_m(w_2, w)] \wedge [R_m(w, w_1) \leftrightarrow R_m(w, w_2)] \tag{D1}$$

$$\forall w : R_m(w, w) \tag{A1}$$

$$\forall w_1, w_2, w_3 : R_m(w_1, w_2) \wedge R_m(w_2, w_3) \rightarrow R_m(w_1, w_3) \tag{A2}$$

$$\forall w_1, w_2 : [\bigwedge_m E_m(w_1, w_2)] \rightarrow w_1 = w_2 \tag{A3}$$

Atomic context terms are evaluated with six valuation relations $V_m \subseteq (CVar \cup \{\bot, \top\}) \times W$. Informally speaking, $V_m(a, w)$ holds between an atomic context term a – intuitively: a name for something – and a context w if and only if w is m-contained in a context that could be named a. Accordingly, $V_m(a, w)$ is read "a *m-applies* to w." Each relation V_m has the following properties: if a m-applies to x then it also m-applies to all contexts y contained in x; $\top$ m-applies to all contexts and $\bot$ m-applies to none:

$$\forall x, y \in W, a \in CVar : V_m(a, x) \wedge R_m(x, y) \rightarrow V_m(a, y) \tag{A4}$$

$$\forall x \in W : V_m(\top, x) \wedge \neg V_m(\bot, x). \tag{A5}$$

It should be remarked that this does not entail that there has to be a context corresponding exactly to a.

Definition 3. *Truth with respect to a model is defined recursively as a relation* $\models$ *holding between a model* $M = \langle\langle W, R\rangle, V\rangle$*, a certain world* $x \in W$*, and a* *context logic formula ϕ. For $(M,x) \models \phi$, we say "ϕ is true in M local to x." We first give the interpretation for the propositional logic connectives:*

$$(M,x) \models \phi \wedge \psi \textit{ iff } (M,x) \models \phi \textit{ and } (M,x) \models \psi \quad \text{[M1]}$$

$$(M,x) \models \phi \vee \psi \textit{ iff } (M,x) \models \phi \textit{ or } (M,x) \models \psi \quad \text{[M2]}$$

$$(M,x) \models \phi \rightarrow \psi \textit{ iff } (M,x) \not\models \phi \textit{ or } (M,x) \models \psi \quad \text{[M3]}$$

$$(M,x) \models \neg\phi \textit{ iff } (M,x) \not\models \phi \quad \text{[M4]}$$

The meaning of positive atomic formulae with atomic context terms is characterised as:

$$(M,x) \models [\top \sqsubseteq_m a] \textit{ for atomic context terms } a \textit{ iff } V_m(a,x) \quad \text{[M5]}$$

The meaning of the sub-context operators is determined with the following rule schema:

$$(M,x) \models [c \sqsubseteq_m d] \textit{ iff for all } y \textit{ with } R_m(x,y), \quad \text{[M6]}$$
$$(M,y) \not\models [\top \sqsubseteq_m c] \textit{ or } (M,y) \models [\top \sqsubseteq_m d]$$

The meaning of the context term operators is determined with the following definitions:

$$(M,x) \models [\top \sqsubseteq_m -c] \textit{ iff } (M,x) \models [c \sqsubseteq_m \bot] \quad \text{[M7]}$$

$$(M,x) \models [\top \sqsubseteq_m c \sqcap d] \textit{ iff } (M,x) \models [\top \sqsubseteq_m c] \textit{ and } (M,x) \models [\top \sqsubseteq_m d] \quad \text{[M8]}$$

$$(M,x) \models [\top \sqsubseteq_m c \sqcup d] \textit{ iff for all } y \textit{ with } R_m(x,y), \quad \text{[M9]}$$
$$(M,y) \not\models [c \sqsubseteq_m \bot] \textit{ or } (M,y) \not\models [d \sqsubseteq_m \bot]$$

We can see that $(M,x) \not\models [\top \sqsubseteq_m \bot]$ for any M and x. And using this we can derive that $[\top \sqsubseteq_m c \sqcup d]$ is equivalent to $[\top \sqsubseteq_m -(-c \sqcap -d)]$:

$$(M,x) \models [\top \sqsubseteq_m -(-c \sqcap -d)] \text{ iff } (M,x) \models [-c \sqcap -d \sqsubseteq_m \bot]$$
$$\text{iff for all } y \text{ with } R_m(x,y), (M,y) \not\models [\top \sqsubseteq_m -c \sqcap -d] \text{ or } (M,y) \models [\top \sqsubseteq_m \bot]$$
$$\text{iff there is no } y \text{ with } R_m(x,y), (M,y) \models [\top \sqsubseteq_m -c] \text{ and } (M,y) \models [\top \sqsubseteq_m -d]$$
$$\text{iff for all } y \text{ with } R_m(x,y), (M,y) \not\models [\top \sqsubseteq_m -c] \text{ or } (M,y) \not\models [\top \sqsubseteq_m -d]$$
$$\text{iff for all } y \text{ with } R_m(x,y), (M,y) \not\models [c \sqsubseteq_m \bot] \text{ or } (M,y) \not\models [d \sqsubseteq_m \bot] \qquad \Box$$

We can show that $[c \sqsubseteq_m d]$ implies $[\top \sqsubseteq_m -c \sqcup d]$. A formula ϕ is *satisfied* in M ($M \models \phi$) iff $(M,x) \models \phi$ holds for some x. A formula ϕ is *valid* or a *tautology* (written: $\models \phi$) iff it is true for any M; ϕ entails another formula ψ iff any for $M \models \phi$ also $M \models \psi$. It follows that the formulae $[\top \sqsubseteq_m c \sqcup -c]$ and $[\top \sqsubseteq_m c] \vee \neg[\top \sqsubseteq_m c]$ – but not the formula $[\top \sqsubseteq_m c] \vee [\top \sqsubseteq_m -c]$ – are tautologies. We call a formula ϕ *satisfiable* iff a model M exists, such that $M \models \phi$. Accordingly, a contextual knowledge base *CKB* is satisfiable iff $\bigwedge_{\phi \in CKB} \phi$ is satisfiable.

6. Tableau Reasoning Procedure

We define a simple tableau reasoning mechanism in the style used by Chagrov *et al.* (1997) and show soundness and completeness. A simple implementation is straight forward, for faster reasoning techniques known from description logic reasoners – in particular, optimisations for handling partial orders – can be applied. A detailed discussion of practical issues is beyond the scope of this article.

Informally, a tableau reasoner works by expanding sets of formulae, called *tableaux*, according to semantically founded rules, either until no further expansion is possible, that is, the tableaux are *saturated*, or until a contradiction that cannot be avoided is found. In the latter case, the algorithm yields "unsatisfiable"; otherwise, the algorithm answers "satisfiable," and the resultant set of tableaux can be interpreted as a model fulfilling the asked question. The reasoning mechanism can be defined as follows.

Definition 4. *A CL-tableau t is a pair of sets of formulae $\langle \Gamma, \Delta \rangle$, where the set Γ contains the formulae which are assumed to be true, whereas Δ contains the formulae which are assumed to be false. A tableau is* saturated *iff it has the following properties for complex formulae:*

$$\phi \wedge \psi \in \Gamma \textit{ then } \phi \in \Gamma \textit{ and } \psi \in \Gamma \qquad \phi \wedge \psi \in \Delta \textit{ then } \phi \in \Delta \textit{ or } \psi \in \Delta \qquad \text{[S1]}$$

$$\phi \vee \psi \in \Gamma \textit{ then } \phi \in \Gamma \textit{ or } \psi \in \Gamma \qquad \phi \vee \psi \in \Delta \textit{ then } \phi \in \Delta \textit{ and } \psi \in \Delta \qquad \text{[S2]}$$

$$\phi \rightarrow \psi \in \Gamma \textit{ then } \phi \in \Delta \textit{ or } \psi \in \Gamma \qquad \phi \rightarrow \psi \in \Delta \textit{ then } \phi \in \Gamma \textit{ and } \psi \in \Delta \qquad \text{[S3]}$$

$$\neg\phi \in \Gamma \textit{ then } \phi \in \Delta \qquad \neg\phi \in \Delta \textit{ then } \phi \in \Gamma; \qquad \text{[S4]}$$

and the following properties for atomic formulae with complex context terms:

$$[c \sqsubseteq_m d] \in \Gamma \textit{ then } [\top \sqsubseteq_m c] \in \Delta \textit{ or } [\top \sqsubseteq_m d] \in \Gamma \qquad \text{[S5]}$$

$$[\top \sqsubseteq_m -c] \in \Gamma \textit{ then } [c \sqsubseteq_m \bot] \in \Gamma \qquad \text{[S6]}$$

$$[\top \sqsubseteq_m -c] \in \Delta \textit{ then } [c \sqsubseteq_m \bot] \in \Delta \qquad \text{[S7]}$$

$$[\top \sqsubseteq_m c \sqcap d] \in \Gamma \textit{ then } [\top \sqsubseteq_m c] \in \Gamma \textit{ and } [\top \sqsubseteq_m d] \in \Gamma \qquad \text{[S8]}$$

$$[\top \sqsubseteq_m c \sqcap d] \in \Delta \textit{ then } [\top \sqsubseteq_m c] \in \Delta \textit{ or } [\top \sqsubseteq_m d] \in \Delta \qquad \text{[S9]}$$

$$[\top \sqsubseteq_m c \sqcup d] \in \Gamma \textit{ then } [c \sqsubseteq_m \bot] \in \Delta \textit{ or } [d \sqsubseteq_m \bot] \in \Delta \qquad \text{[S10]}$$

A tableau is called disjoint *iff the sets Γ and Δ are disjoint: $\Gamma \cap \Delta \neq \emptyset$.*

From the definition of saturation we can directly derive corresponding saturation rules for a non-deterministic tableaux algorithm. By induction on the number of occurrences of variables in formulae in the sets Γ and Δ, we can show that this algorithm terminates for any finite tableau $t = \langle \Gamma, \Delta \rangle$. However, the saturation rules alone do not yield a complete reasoning mechanism for context logics.

Definition 5. *A* Hintikka system *in context logics is a pair* $H = \langle T, S\rangle$ *where* $S = \langle S_{\text{who}}, S_{\text{when}}, S_{\text{where}}, S_{\text{what}}, S_{\text{how}}, S_{\text{why}}\rangle$ *and the* S_m *are binary relations on* T *that are reflexive and transitive, and antisymmetric up to* m*-equivalence and* T *is a non-empty set of disjoint, saturated tableaux fulfilling the following three properties.*
For tableaux $t = \langle\Gamma, \Delta\rangle, t' = \langle\Gamma', \Delta'\rangle$ *with* $t, t' \in T$ *and* $S_m(t, t')$ *holds that*

$$\textit{for all atomic formulae } \phi : \phi \in \Gamma \textit{ implies } \phi \in \Gamma'. \qquad \text{[H1]}$$

For any tableau $t = \langle\Gamma, \Delta\rangle$ *with* $[c \sqsubseteq_m d] \in \Delta$*, there is a tableau* $t' = \langle\Gamma', \Delta'\rangle$ *with* $S_m(t, t')$ *such that*

$$[\top \sqsubseteq_m c] \in \Gamma' \textit{ and } [\top \sqsubseteq_m d] \in \Delta'. \qquad \text{[H2]}$$

For any tableau $t = \langle\Gamma, \Delta\rangle$ *with* $[\top \sqsubseteq_m c \sqcup d] \in \Delta$*, there is a tableau* $t' = \langle\Gamma', \Delta'\rangle$ *with* $S_m(t, t')$ *such that*

$$[c \sqsubseteq_m \bot] \in \Gamma' \textit{ and } [d \sqsubseteq_m \bot] \in \Gamma'. \qquad \text{[H3]}$$

$H = \langle T, S\rangle$ *is a* CL-Hintikka system for a tableau $t = \langle\Gamma, \Delta\rangle$ *if there is a tableau* $t' = \langle\Gamma', \Delta'\rangle$ *in* T *such that* $\Gamma \subseteq \Gamma'$ *and* $\Delta \subseteq \Delta'$*.*

The question whether a query sentence κ follows from a context knowledge base CKB can be formulated as the question whether the tableau $t = \langle CKB, \{\kappa\}\rangle$ is unsatisfiable: a Hintikka system for t exists if and only if $CKB \not\models \kappa$. We show a result for soundness and weak completeness:[5] there is a model M for t so that $M \models \phi$ for every $\phi \in \Gamma$ and $M \not\models \psi$ for every $\psi \in \Delta$, iff there is a CL-Hintikka system for t. It is clear that the proof can be restricted to those formulae $\phi \in CFor$ that (a) are constructed using only context variables in t and (b) have a lower or the same number of variable occurrences than the formulae in t. We accordingly define the set $\Sigma \subset CFor$ as the set of formulae that can be constructed from t in this way. The intuition behind the proof is to identify tableaux $t \in T$ with worlds $w \in W$. The set Γ of the tableau is constructed to match the set of sentences true in w, and the set Δ accordingly to the set of sentences false in w. The relations S_m connecting tableaux are matched to the relations R_m connecting worlds. We first show soundness: if $CKB \models \kappa$ does not hold, our mechanism does not return "unsatisfiable" for the query $t = \langle CKB, \{\kappa\}\rangle$.

Proof. ($\Rightarrow$, soundness) Consider, we have a model $M = \langle\langle W, R\rangle, V\rangle$, so that $M \models \phi$ for all $\phi \in CKB$ and $M \not\models \kappa$. We show that, in this case, we can also find a Hintikka system for the initial tableau $t = \langle CKB, \{\kappa\}\rangle$.

5. We can decide unsatisfiability for a set of formulae. But the method is not complete in the strong sense that all consequences of a set of formulae can be derived using the mechanism.

Given a model $M = \langle\langle W, R\rangle, V\rangle$ for t, we construct a corresponding Hintikka system $H = \langle T, S\rangle$ with

$$T = \{\langle\Gamma_x, \Delta_x\rangle \mid \forall\phi \in \Sigma : \phi \in \Gamma_x \text{ iff } (M, x) \models \phi, \text{ and } \phi \in \Delta_x \text{ iff } (M, x) \not\models \phi\}$$

$$S_m(\langle\Gamma_x, \Delta_x\rangle, \langle\Gamma_y, \Delta_y\rangle) \text{ iff } R_m(x, y)$$

We have to show that $\langle T, S\rangle$ actually is a Hintikka system, i.e., that the tableaux $\langle\Gamma_x, \Delta_x\rangle \in T$ thus defined are disjoint and saturated and that the three conditions [H1]-[H3] hold.

Disjointness follows from the fact that, for any world $x \in W$ and any formula $\phi \in \Sigma$, either $(M, x) \models \phi$ and $\phi \in \Gamma_x$, or $(M, x) \not\models \phi$ and $\phi \in \Delta_x$.

That each t is saturated can be seen by checking the correspondence between each of the saturation rules and the definition of the semantic relation $\models$. In the case of $[c \sqsubseteq_m d] \in \Gamma_x$, for instance, the saturation rule [S5] demands that $[\top \sqsubseteq_m c] \in \Delta_x$ or $[\top \sqsubseteq_m d] \in \Gamma_x$. This is given, since the corresponding definition of $\models$ ensures that $(M, x) \models [c \sqsubseteq_m d]$ iff for all y with $R_m(x, y)$ $(M, y) \not\models [\top \sqsubseteq_m c]$ or $(M, y) \models [T \sqsubseteq_m d]$. Since $R_m(x, x)$ [A1] this also holds for x and thus we also have either $[\top \sqsubseteq_m c] \in \Delta_x$, or $[\top \sqsubseteq_m d] \in \Gamma_x$. Similarly, we can directly show saturation for $[\top \sqsubseteq_m -c]$ and $[\top \sqsubseteq_m c \sqcap d]$, whether they are in Γ or Δ, and saturation for the case $[\top \sqsubseteq_m c \sqcup d] \in \Gamma$, as well as saturation for formulae constructed using the propositional logic connectives.

We can now prove that the rules [H1]-[H3] also hold for H. The first rule follows from the definition of $(M, x) \models [c \sqsubseteq_m d]$ together with the transitivity of R_m: if $[c \sqsubseteq_m d] \in \Gamma_x$ and thus $(M, x) \models [c \sqsubseteq_m d]$, then for all $y, R_m(x, y)$ $(M, y) \not\models [\top \sqsubseteq_m c]$ or $(M, y) \models [T \sqsubseteq_m d]$. Since $R_m(x, y)$ and $R_m(y, y')$ entails $R_m(x, y')$ for all y', it follows that also $(M, y') \not\models [\top \sqsubseteq_m c]$ or $(M, y') \models [T \sqsubseteq_m d]$ and thus $(M, y) \models [c \sqsubseteq_m d]$ and $[c \sqsubseteq_m d] \in \Gamma_y$.

For the second rule, assume $\langle\Gamma_x, \Delta_x\rangle \in T$ with $[c \sqsubseteq_m d] \in \Delta_x$. In this case, $(M, x) \not\models [c \sqsubseteq_m d]$ holds. Then we know that there is $y \in W$ with $R_m(x, y)$ and $(M, y) \models [\top \sqsubseteq_m c]$ and $(M, y) \not\models [\top \sqsubseteq_m d]$; and accordingly, there must be a tableau $\langle\Gamma_y, \Delta_y\rangle \in T$ with $S_m(\langle\Gamma_x, \Delta_x\rangle, \langle\Gamma_y, \Delta_y\rangle)$ with $[\top \sqsubseteq_m c] \in \Gamma_y$ and $[\top \sqsubseteq_m d] \in \Delta_y$.

For the third rule, we analogously assume $\langle\Gamma_x, \Delta_x\rangle \in T$ with $[T \sqsubseteq_m c \sqcup d] \in \Delta_x$. This entails that $(M, x) \not\models [T \sqsubseteq_m c \sqcup d]$ and thus that there is $y \in W$ with $R_m(x, y)$ and $(M, y) \models [c \sqsubseteq_m \bot]$ and $(M, y) \models [d \sqsubseteq_m \bot]$. The corresponding tableau $\langle\Gamma_y, \Delta_y\rangle \in T$ with $S_m(\langle\Gamma_x, \Delta_x\rangle, \langle\Gamma_y, \Delta_y\rangle)$ fulfils the requirement $[c \sqsubseteq_m \bot] \in \Gamma_y$ and $[d \sqsubseteq_m \bot] \in \Gamma_y$. □

We can now give a result for weak completeness. We have to show: if our mechanism did not return "unsatisfiable" for the query $t = \langle CKB, \{\kappa\}\rangle$ then $CKB \models \kappa$ does not hold.

Proof. ($\Leftarrow$, weak completeness) Consider, we found a Hintikka system for the initial tableau $t = \langle CKB, \{\kappa\}\rangle$. We show that, in this case, we can construct a model $M = \langle\langle W, R\rangle, V\rangle$, so that $M \models \phi$, for all $\phi \in CKB$ and $M \not\models \kappa$.

We show that we can interpret a given Hintikka system $H = \langle T, S\rangle$ for a tableau t as a frame and construct a model $M = \langle H, V\rangle$ by defining V for any context variable a in formulae in $CBK \cup \kappa$ and any tableau $\langle\Gamma, \Delta\rangle$ as follows:

$$V_m(a, \langle\Gamma, \Delta\rangle) \text{ iff } [\top \sqsubseteq_m a] \in \Gamma$$

By induction on the number of occurrences of variables in a formula ϕ, we show that for any tableau $\langle\Gamma, \Delta\rangle \in T$

$$\phi \in \Gamma \text{ implies } (M, \langle\Gamma, \Delta\rangle) \models \phi$$

$$\phi \in \Delta \text{ implies } (M, \langle\Gamma, \Delta\rangle) \not\models \phi$$

The basis of the induction are the formulae $\phi = [\top \sqsubseteq_m a]$ for atomic context terms a occurring in formulae in Σ. That the two properties hold in this case immediately follows from the definition of V_m. Similarly, for the other formulae with a number of occurrences of variables $\mathrm{varN}(\phi) \leq 1$.

For formulae $[c \sqsubseteq_m d]$ and $[\top \sqsubseteq_m -c]$, $[\top \sqsubseteq_m c \sqcup d]$, $[\top \sqsubseteq_m c \sqcap d]$ with arbitrarily complex context terms c, d, the induction has to follow the structure of the context terms. The induction assumption in this case is that the properties hold for the positive and negative atoms of c and d, that is, for the formulae $[\top \sqsubseteq_m c]$, $[\top \sqsubseteq_m d]$, $[c \sqsubseteq_m \bot]$, and $[d \sqsubseteq_m \bot]$, for whom *varN* is determined solely by the structure of c and d, respectively.

For $[c \sqsubseteq_m d]$, first suppose $[c \sqsubseteq_m d] \in \Gamma$ for a tableau $\langle\Gamma, \Delta\rangle$. By the saturation rules $[\top \sqsubseteq_m c] \in \Delta$ or $[\top \sqsubseteq_m d] \in \Gamma$. And by the first rule, this property holds also for all tableau $\langle\Gamma', \Delta'\rangle$ for which $S_m(\langle\Gamma, \Delta\rangle, \langle\Gamma', \Delta'\rangle)$ holds. By the induction assumption, this implies $(M, \langle\Gamma', \Delta'\rangle) \not\models [\top \sqsubseteq_m c]$ or $(M, \langle\Gamma', \Delta'\rangle) \models [\top \sqsubseteq_m d]$ for all $\langle\Gamma', \Delta'\rangle$ with $S_m(\langle\Gamma, \Delta\rangle, \langle\Gamma', \Delta'\rangle)$. And this in turn implies by the definition of $\models$ that $(M, \langle\Gamma, \Delta\rangle) \models [c \sqsubseteq_m d]$.

If $[c \sqsubseteq_m d] \in \Delta$, then by the second rule there is a tableau $\langle\Gamma', \Delta'\rangle$ with $S_m(\langle\Gamma, \Delta\rangle, \langle\Gamma', \Delta'\rangle)$ such that $[\top \sqsubseteq_m c] \in \Gamma'$ and $[\top \sqsubseteq_m d] \in \Delta'$. By the induction assumption, this entails that $(M, \langle\Gamma', \Delta'\rangle) \models [\top \sqsubseteq_m c]$ and $(M, \langle\Gamma', \Delta'\rangle) \not\models [\top \sqsubseteq_m d]$ and thus $(M, \langle\Gamma, \Delta\rangle) \not\models [c \sqsubseteq_m d]$ by the definition of $\models$.

For the cases of $[\top \sqsubseteq_m -c]$ and $[\top \sqsubseteq_m c \sqcap d]$ in Γ or Δ, the saturation rules directly correspond to the definition of $\models$. The case of $[\top \sqsubseteq_m c \sqcup d] \in \Gamma$ is similar to that of $[c \sqsubseteq_m d] \in \Gamma$: since $[\top \sqsubseteq_m c \sqcup d]$ is an atomic formula, $[\top \sqsubseteq_m c \sqcup d] \in \Gamma'$ for all tableaux $\langle\Gamma', \Delta'\rangle$ for which $S_m(\langle\Gamma, \Delta\rangle, \langle\Gamma', \Delta'\rangle)$ [H1]. By [S10], we also know that $[c \sqsubseteq_m \bot] \in \Delta$ or $[d \sqsubseteq_m \bot] \in \Delta$ holds in all these tableaux and in $\langle\Gamma, \Delta\rangle$ itself. By the induction assumption and [M9], we see that also $(M, \langle\Gamma, \Delta\rangle) \models [\top \sqsubseteq_m c \sqcup d]$ holds. The case of $[\top \sqsubseteq_m c \sqcup d] \in \Delta$ follows by the correspondence between the rule [H3] and the definition [M9].

For formulae ϕ constructed using the propositional connectives $\neg, \wedge, \vee, \rightarrow$, the induction assumption is that the properties are given for formulae ψ and χ. The proof in this case is obvious, we show the example of $\phi = \psi \wedge \chi \in \Gamma$. If the formula $\psi \wedge \chi$ is in Γ, then according to the saturation rules: $\psi \in \Gamma$ and $\chi \in \Gamma$. Using the induction assumption, this implies that $(M, \langle \Gamma, \Delta \rangle) \models \psi$ and $(M, \langle \Gamma, \Delta \rangle) \models \chi$, and this in turn entails $(M, \langle \Gamma, \Delta \rangle) \models \phi$. $\square$

7. Conclusions and Future Works

We introduced a new logical language for formulating context ontologies as part of development, configuration, or personalisation of context-aware applications. We sketched how the logical language can be connected with an infrastructure and application model for context-aware computing applications. We showed that the 5W1H context model, which represents the six domains of social, temporal, spatial, taxonomic, state-related, and task-related knowledge in a unifying syntax, can support a context-oriented, mereologic perspective on the representation of context. We argued that the object-oriented view is not well suited for representing context ontologies and context models and showed two main weak points of current ontology-based approaches to context-aware computing. Current ontologies lack semantic clarity with respect to core concepts, such as spatial containment, and use expressive logical frameworks, which lead to unnecessary complexity in reasoning. It is clear that reliable and robust context-aware applications, that is, applications that can be used fast and flexibly in everyday life by non-expert users, require a sound and complete logical framework with a simple syntactic interface. The simple syntax of the context logic and its unifying perspective on six domains of context were developed as a step into this direction. As a first practical result, we showed that the language is sufficiently expressive to capture an example from previous context-aware applications. In the formal part, proofs for soundness and completeness for reasoning about the six domains were given.

In future works, we plan to implement a context logic reasoning engine and a graphical interface for editing 5W1H containment hierarchies. An ongoing focus of research is to extend the language with additional operators beyond simple containment hierarchies, which realise only the dimension of partial representations (Benerecetti *et al.*, 2000). We intend to introduce approximate representations using granularity relations (Schmidtke *et al.*, 2007) and perspectival representations based on reference systems (Eschenbach, 1999). Reference systems have been shown to be not only important for spatial and temporal reasoning but also for reasoning about agents in organisational structures (Zacarias *et al.*, 2007). Also, we are currently working on a user study involving developers of context-aware applications.

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