An Effective Approach for Wavelet Lifting Based on Filter Optimization and Median Operator

Do Quan, Yo-Sung Ho

Department of Information and Communications, Gwangju Institute of Science and Technology (GIST), 261 Cheomdan-gwagiro, Buk-gu, Gwangju 500-712, South Korea

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ABSTRACT: In JPEG2000, the Cohen–Daubechies–Feauveau (CDF) 9/7-tap wavelet filter implemented by using the conventional lifting scheme has two problems. The first problem is that the filter coefficients are remaining complex; second, the conventional lifting scheme ignores image edges in the coding process. In this article, we propose an effective wavelet lifting scheme to solve these problems. For this purpose, we design the optimal 9/7-tap wavelet filters in two steps. In the first step, we select the appropriate filter coefficients; in the second step, we employ a median operator to consider image edges. Experimental results from using the median lifting scheme and combining filter optimization and median lifting show that our proposed methods outperform the well-known CDF 9/7-tap wavelet filter of JPEG2000 on edge-dominant images. Int J Imaging Syst Technol, 20, 359–366, 2010; Published online in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/ima.20255

Key words: wavelet transforms; image coding; lifting scheme; filter design; filter optimization; JPEG2000

I. INTRODUCTION

Grossmann and Morlet (1984) proposed wavelets, which are known as the first-generation wavelets implemented by using the convolution scheme. We can use the lifting scheme as the second-generation wavelets (Sweldens, 1996; Daubechies and Sweldens, 1998). The lifting scheme is known to outperform the convolution scheme in various ways: it requires less computation and a smaller memory, it can more easily produce integer-to-integer wavelet transform, and it is both invertible and revertible.

In JPEG2000, the Cohen–Daubechies–Feauveau (CDF) 9/7-tap wavelet filter was implemented for lossy image compression by using the lifting scheme. On the basis of the CDF 9/7-tap wavelet filter using the lifting scheme, scientists have proposed several other wavelet filters that have different coefficients. In the designing process, they use a parameter α to find these coefficients.

Several 9/7-tap wavelet filters have different coefficients. Daubechies and Sweldens (1998) used the lifting scheme and the factoring method with the irrational number $\alpha = -1.586134342\ldots$ to find the coefficients of the CDF 9/7-tap wavelet filter. Similarly, Guangjun et al. (2001) proposed a simple 9/7-tap wavelet filter using the lifting scheme and another parameter $\alpha = -1.5$. Compared with the CDF 9/7-tap wavelet filter, their filter gives similar performance in terms of peak signal-to-noise ratio (PSNR), but α is simplified into a rational number. Liang et al. (2003) used a temporary parameter $t = 1.25$ to generate a filter (for the CDF 9/7-tap wavelet filter, $t = 1.230174\ldots$). This filter performs at the same level but its parameter t is less complex than that of the CDF 9/7-tap wavelet filter.

Although the wavelet filters proposed by Guangjun et al. (2001) and Liang et al. (2003) perform equally well or slightly better than the CDF 9/7-tap wavelet filter, their performances are not optimal. The main concern is whether there is any other α value and its corresponding coefficients of wavelet filters that can provide better performance with less complex coefficients than those of the CDF 9/7-tap wavelet filter.

To enhance the performance of the conventional lifting scheme, scientists have proposed many methods. These methods include several adaptive lifting schemes (Taubman, 1999; Piella and Heijmans, 2002; Sole and Salembier, 2004; Hattay et al., 2005; Hou and Verdes, 2006). In the following works, the authors used ''directional'' and ''orientation'' terms to propose the methods. Multidirection provides a significant efficiency for image compression and image processing, not only in the horizontal direction but also in the vertical direction. Velisavljevic et al. (2006) proposed novel anisotropic transforms for images that used separable filtering in many directions, not only horizontal and vertical. Chang et al. (2005) applied a directional quincunx lifting scheme in JPEG2000 to obtain a better coding performance. Furthermore, Ding et al. (2007) and Wu and Li (2006) proposed a directional lifting scheme, a combination of the conventional lifting scheme of JPEG2000 with the concept of H.264 intra-prediction to obtain doubled coding efficiency on images with rich orientation features. After dividing an image into blocks, Ding et al. and Wu and Li first used the energy function to determine the direction of each block. They

Correspondence to: Do Quan; e-mail: viequando@gist.ac.kr

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subsequently used intra-prediction to encode each block. Besides, Liu and Ngan (2008), in his proposed weighted adaptive lifting scheme, used weighted functions which predict different directions and which code images with rich orientation features. Gerek and Cetin (2006) used an orientation adaptive lifting scheme based on the 2D orientation-adaptive prediction filter. Moreover, Chang and Girod (2007) proposed a direction-adaptive discrete wavelet transform (DWT) implemented by using the lifting scheme for the anisotropic image model.

Both the first-generation and second-generation wavelets unfortunately disregard image edges in image coding operation. Some authors have proved that images need to be considered to enhance the coding efficiency (Po and Do, 2006). In addition, image edges are affected by the ringing artifacts during the coding process (Gerek and Centin, 2000). To solve these problems, we propose methods that consider image edges to enhance the coding efficiency and eliminate the ringing artifacts.

In this article, we will propose methods for coding edge-dominant images. We first design the optimal 9/7-tap wavelet filters by choosing an α value from the whole possible range of α. This α value will result in the optimal performance. Because median lifting, being a nonlinear operation, yields much better image quality for edge-dominant images (Jansen and Oonincx, 2005), we propose a new lifting scheme which uses a median operator. We will then obtain an efficient lifting scheme based on filter optimization and median operation.

The rest of this article is organized as follows. Section II describes a numerical analysis and the conventional lifting scheme. Section III describes both methods to design the optimal wavelet filters and our proposed lifting scheme. Section IV presents experimental results and analysis. Section V concludes this article.

II. BASIS OF WAVELET FILTER DESIGN AND CONVENTIONAL LIFTING SCHEME

In this section, we present the basis of wavelet filter design and explain how to construct a 9/7-tap wavelet filter. In addition, we briefly describe the conventional lifting scheme that has been used in many designs of wavelet filters including the CDF 9/7-tap wavelet filter of JPEG2000 and the wavelet filter of Guangjun's method. Through the description, we present the existing problems of the conventional lifting scheme and the CDF 9/7-tap wavelet filter. Finally, we present our solutions to solve these problems.

A. Basis of Wavelet Filter Design.

In wavelets, the forward transform uses both a low-pass filter, h, and a high-pass filter, g, followed by subsampling. In the z-domain, h can be expressed as in the work by Daubechies and Sweldens (1998):

$$h(z) = \sum_{k=k_{\min}}^{k_{\max}} h_k z^{-k}, \tag{1}$$

where $k_{\min}$ and $k_{\max}$ are, respectively, the smallest and the greatest integer numbers for which h_k is a nonzero coefficient.

The functions of h and g in the ω-domain are defined as in the work by Cohen et al. (1992):

$$H(\omega) = h_0 + 2\sum_{n=1}^{L_1} h_n \cos\; n\omega, \tag{2}$$

$$G(\omega) = g_0 + 2\sum_{n=1}^{L_2} g_n \cos\; n\omega, \tag{3}$$

where $L_1 = 4$ and $L_2 = 3$ in the CDF 9/7-tap wavelet filter.

If $H(\omega)$ and $G(\omega)$ construct a biorthogonal wavelet, the normalized condition must be satisfied

$$H(0) = \sqrt{2} \quad \text{and} \quad G(0) = \sqrt{2}. \tag{4}$$

Combining Eq. (4) with Eqs. (2) and (3), we get

$$\begin{cases} h_0 + 2(h_1 + h_2 + h_3 + h_4) = \sqrt{2} \\ g_0 + 2(g_1 + g_2 + g_3) = \sqrt{2} \end{cases}. \tag{5}$$

Substituting $\omega = \pi$ into $H(\omega)$ of Eq. (2) and into $G(\omega)$ of Eq. (3), we obtain

$$h_0 + 2(-h_1 + h_2 - h_3 + h_4) = 0, \tag{6}$$

$$g_0 + 2(-g_1 + g_2 - g_3) = 0. \tag{7}$$

Taking the second derivative of Eq. (2), we get

$$2(-g_1 + 4g_2 - 9g_3) = 0. \tag{8}$$

A polyphase matrix $P_a(z)$ is presented by

$$\begin{aligned} P_a(z) &= \begin{bmatrix} h_e(z) & g_o(z) \\ h_o(z) & g_e(z) \end{bmatrix} \\ &= \begin{bmatrix} h_0 + h_2(z+z^{-1}) + h_4(z^2+z^{-2}) & g_1(1+z^{-1}) + g_3(z+z^{-2}) \\ h_1(z+1) + h_3(z^2+z^{-1}) & -g_0 - g_2(z+z^{-1}) \end{bmatrix}. \end{aligned} \tag{9}$$

where h_e and g_e are even coefficients; h_o and g_o are the odd coefficients.

The polyphase matrix $P_a(z)$ can be factorized into five elementary matrices as in the work by Daubechies and Sweldens (1998):

$$\begin{aligned} P_a(z) &= \begin{bmatrix} 1 & \alpha(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta(1+z) & 1 \end{bmatrix} \\ &\quad \times \begin{bmatrix} 1 & \gamma(1+z^{-1}) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \delta(1+z) & 1 \end{bmatrix} \begin{bmatrix} \xi & 0 \\ 0 & \frac{1}{\xi} \end{bmatrix}. \end{aligned} \tag{10}$$

Using Eq. (5) to Eq. (8) and equating an equivalence of $P_a(z)$ parameters in Eqs. (9) and (10), we have:

$$\begin{cases} \beta = -\frac{1}{4(1+2\alpha)^2} \\ \gamma = -\frac{(1+2\alpha)^2}{1+4\alpha} \\ \delta = \frac{1}{16}\left(4 + \frac{1-8\alpha}{(1+2\alpha)^2} - \frac{2}{(1+2\alpha)^3}\right) \\ \xi = \frac{2\sqrt{2}(1+2\alpha)}{1+4\alpha} \end{cases}. \tag{11}$$

In the CDF 9/7-tap wavelet filter, the prediction and updating and normalized parameters are represented by $\alpha = -1.586134\ldots$, $\beta = -0.05298\ldots$, $\gamma = 0.882911\ldots$, $\delta = 0.443506\ldots$, and $\xi = 1.230174\ldots$ (ITU-T T.800, 2002).

In our proposed methods, α is examined in the $(-\infty, +\infty)$ range to find the optimal value for the 9/7-tap wavelet filters in terms of complexity of filter coefficients and performance. However, in practice, we limit the range of α value by calculating the statistical distribution of the optimal α. For each α, we determine the $(\beta, \gamma, \delta, \xi)$ set using Eq. (11), we compute the coefficients of h and g using Eqs. (9) and (10), and finally, we get the optimal α based on the maximum value of PSNR, i.e., $PSNR_{max}$, of the corresponding 9/7-tap wavelet filter for each experimental image and compression ratio.

B. Conventional Lifting Scheme. This subsection briefly presents the conventional lifting scheme, as it has been used in many designs of wavelet filters, including the CDF 9/7-tap wavelet filter adopted in JPEG2000 and the wavelet filter of Guangjun's method. The conventional lifting scheme, which is a space-domain construction of biorthogonal wavelets proposed by Sweldens (1996), consists of four steps: splitting, prediction, updating, and normalization. To apply lifting scheme to image processing, we divide an image s into odd s_o and even s_e columns or rows in the splitting step (Hampson and Pesquet, 1996; Jansen and Oonincx, 2005). In the prediction step, we obtain the detail components by using the following formula

$$h(m,n) = s_o(m,n) - \sum_i p_i s_e(m, n+i), \tag{12}$$

where p_i is the prediction parameter. In the updating step, we calculate the approximation components as follows:

$$l(m,n) = s_e(m,n) + \sum_j u_j h(m, n+j), \tag{13}$$

where u_j is the updating parameter. These steps can be reiterated to create the complete set of DWT scaling and that of wavelet coefficients. The final step is normalization. Consequently, p_i, u_j, and the normalized parameters are equivalent to the $(\alpha, \beta, \gamma, \delta, \xi)$ set mentioned in the previous subsection.

Based on Eqs. (12) and (13), the conventional lifting scheme directly uses the splitting data. This means that the scheme considers neither the neighboring elements nor image edges, even though image edges are affected by ringing artifacts during the coding process. If we consider image edges in the lifting scheme, we can obtain a better performance.

III. PROPOSED METHODS

In this section, the aim of our proposed wavelet filter design is to simplify the wavelet coefficients. In other words, the optimal wavelet filters have rational coefficients, unlike the CDF 9/7-tap wavelet filter which has irrational coefficients. First, we propose an optimization method for wavelet filters. Then, we employ a median operator to consider image edges and obtain a median lifting scheme. Finally, we combine the filter optimization and the median lifting to double the coding efficiency.

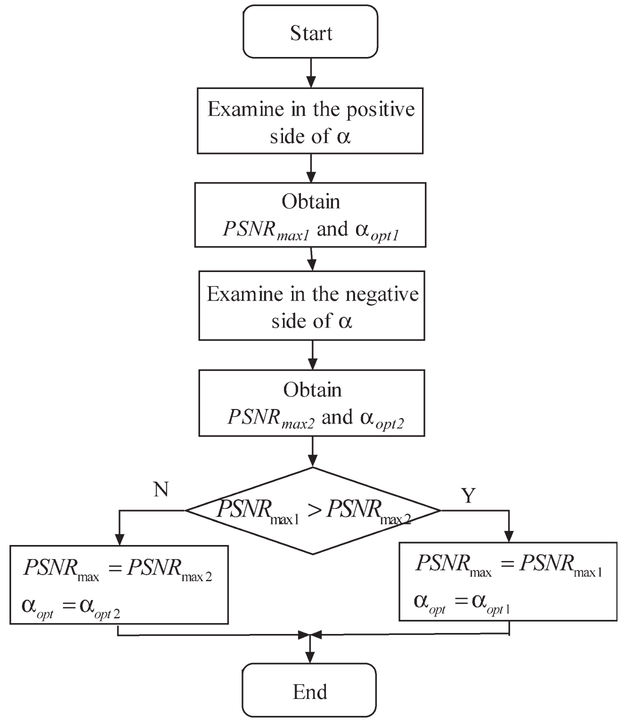

Figure 1. Algorithm to find $PSNR_{max}$ and the optimal α.

A. Optimal Wavelet Filters. From Eq. (11), as β, γ, δ, and ξ depend on α, we can get different β, γ, δ, and ξ values by changing α in the $(-\infty, +\infty)$ range. After finding $(\alpha, \beta, \gamma, \delta, \xi)$ set, the coefficients of the 9/7-tap wavelet filters are computed from Eqs. (9) and (10). For each filter, we find the corresponding PSNR value. Consequently, α has the optimal value when PSNR value is maximal, i.e., $PSNR_{max}$.

In designing optimal wavelet filters, we draw a flowchart of our proposed method as shown in Figure 1. First, we consider both negative and positive values of α. For each side, we can find the local $PSNR_{max}$ and the locally optimal α. Then, we compare the local $PSNR_{max}$ from each side. We select the higher $PSNR_{max}$ as the final $PSNR_{max}$ and we call its corresponding α as the final optimal α, denoted as α_{opt}. Using α_{opt}, we obtain filter coefficients of the 9/7-tap wavelet filter. The shape of a PSNR curve for each 9/7-tap wavelet filter is a concave function of α, as shown in Figure 2. Our experiments in Section IV will confirm this shape.

To reduce the time spent in searching the optimal α, we use a bisection algorithm, as shown in Table I. We have α at two initial positions: left (L) and right (R). Then, we compare the PSNR value at middle (M) position with the PSNR values of (L) and (R). Finally, we use (M) to update (R) or (L). We reiterate this process until the PSNR value reaches its maximum value.

After selecting the optimal α, we substitute this α into Eq. (11) and use our proposed algorithm to obtain other parameters such as β, γ, δ, and ξ. From Eqs. (9) and (10), we obtain the coefficients of the 9/7-tap wavelet filters. Then, we apply the optimal $(\alpha, \beta, \gamma, \delta, \xi)$ set to the median lifting scheme. We describe this process in the next subsection.

B. Median Lifting Scheme. In image coding, the conventional lifting scheme does not consider image edges. In this article, we replace the conventional lifting scheme with the median lifting

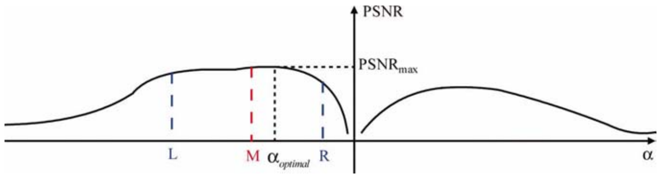

Figure 2. Shape of PSNR as a concave function of α. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

scheme and use the optimal α. This subsection describes the median lifting scheme in detail.

The *N*th-order median filter is defined by:

$$\text{median}_N(s(i,k)) = \text{median}\left(\left\{s\left(i,k-\left\lfloor\frac{N}{2}\right\rfloor\right),...,s\left(i,k+\left\lfloor\frac{N-1}{2}\right\rfloor\right)\right\}\right). \quad (14)$$

To show the efficiency of median filtering for handling image edges, we extract nine pixel values from the chin of GRANDMA image (row 313, columns 130–138). Then, we take the median and mean operations. While the mean filtering matches the wrong edges, we obtain the exact real edges by using the median operation. Figure 3 shows the result in detail.

In proving the efficiency of median filtering for handling image edges by mathematics, some mathematical statisticians have successfully applied various models of median filters for images with edges (Arias-Castro and Donoho, 2009). We know that median filter is nonlinear. Existing works showed the advantages of nonlinear filters (Hampson and Pesquet, 1998; Jansen and Oo護incx, 2005; Arias-Castro and Donoho, 2009).

The median lifting scheme consists of four steps: splitting, prediction, updating, and normalization. The splitting step is similar to that of the conventional lifting scheme. The prediction step is represented by

$$h(m,n) = s_o(m,n) - \sum_i p_i \text{median}_N(s_e(m,n+i)), \quad (15)$$

where p_i is the prediction parameter. We calculate the median of $s_e(m, n + i)$ and its $N - 1$ neighboring even elements. Next, the updating step is represented by

$$l(m,n) = s_e(m,n) + \sum_j u_j \text{median}_M(h(m,n+j)), \quad (16)$$

where u_j is the updating parameter and *M* denotes the order of the median updating filter. These above steps can be reiterated to create the complete set of DWT scaling and that of wavelet coefficients. The (p_i, u_j) set is equivalent to the optimal (α, β, γ, δ) set when we

Table I. Bisection algorithm

1: $s \leftarrow$ PSNR (L);
2: M ← (L + R)/2;
3: If PSNR (M) > s
L ← M;
else
R ← M;
4: Return to step 2 unless R – L is small enough;

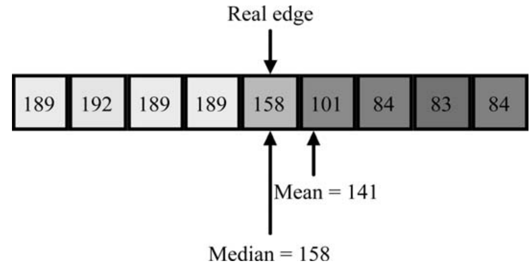

Figure 3. Median operation for handling edge.

apply the optimal (α, β, γ, δ) set to a combination of filter optimization and median lifting. In the conventional lifting scheme, the (p_i, u_j) set is equivalent to the (α, β, γ, δ) set, which is used in the CDF 9/7-tap wavelet filter of JPEG2000. The last step is the normalization step and it uses ξ parameter.

We use prediction, updating, and normalization parameters in similar ways as how other authors have used them. When designing the optimal wavelet filters in the decoding process as the combination of filter optimization and median lifting, we determine these parameters. Hence, we can say that we encode and decode images using the identical lifting scheme.

The median filtering is smoothing (Gonzalez and Woods, 2008). It achieves some fixation in any type of problem due to distortion introduced in the transformation of the original information to the wavelet domain. Some mathematicians and statisticians have proven that the median filtering handles image edges (Arias-Castro and Donoho, 2009). Compared with the conventional lifting scheme, the combination of the median operator and lifting scheme is more robust because it is a nonlinear lifting scheme which considers the neighboring elements. It is also robust for image denoising (Jansen and Ooníncx, 2005). Moreover, using both primal and dual lifting steps, it produces much better image quality when the target image contains complex edges. Therefore, through using the median operator as well as the combination of filter optimization and median lifting, we can obtain improved results for edge-dominant images. Both the mathematical formulations in Arias-Castro and Donoho's work and our experimental results confirm this statement.

IV. RESULTS

In this section, we describe experimental results for several test images that are edge-dominant images. We now describe our experimental results in detail.

A. Distribution of Optimal α. As explained in Section II, to find the optimal wavelet filter, we examine α in the $(-\infty, +\infty)$ range. We obtain a distribution of the optimal α for PSNR$_{max}$ with α step size 0.1. By using this distribution, we limit the examination range of α. We calculate PSNR at four rates: 0.125, 0.25, 0.5, and 1 bpp. The statistics of the optimal α are presented in Figure 4.

B. The Performance of Different Rates. We confirm PSNR curve as a concave function of α, which is used under the assumption in Section III: the shape of PSNR curve for each 9/7-tap wavelet filter is a concave function of α. We only sketch α values in the negative side at four rates and ignore α values in the positive side since the optimal α normally lied on the negative side of α. The shapes of PSNR are expressed in Figure 5 for two images: LYNDA and GRANDMA. These two images illustrate higher PSNR values

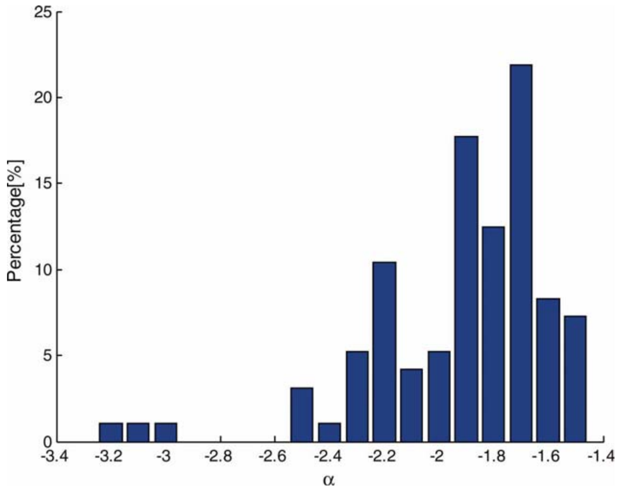

Figure 4. Distribution of the optimal α for $PSNR_{max}$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

and result in a better visual quality. Other test images show similar improvements. From Figure 5, we can find the $PSNR_{max}$ and α_{opt} using the bisection algorithm, as discussed in Section III.

C. Comparison of Different Methods. To compare the coding performances, we obtain the PSNR gains of two proposed methods and those of two reference methods. The two proposed methods are the median lifting scheme and the combination of filter optimization and median lifting, while the two reference methods are the conventional lifting scheme used in the CDF 9/7-tap wavelet filter of JPEG2000 and Guangjun's method. We define the PSNR gain by using the following formula

$$\Delta PSNR_{PR} = PSNR_{\text{Proposed method}} - PSNR_{\text{Reference method}}. \quad (17)$$

The conventional lifting scheme is used in the CDF 9/7-tap wavelet filter of JPEG2000. Besides, Guangjun et al. (2001) designed a wavelet filter and obtained a simpler α; however, the performance was similar to that of the conventional lifting scheme in the CDF 9/7-tap wavelet filter of JPEG2000. Table II shows these performances. As discussed in Subsection IV.A, we first examine the range of α to obtain the α_{opt}; we then use this α_{opt} for the median lifting scheme. As a result, we obtain the median lifting scheme and the combination of filter optimization with median lifting.

As shown in Table II, the optimal α lies between −3.2 and −1.52. However, it lies between −1.7 and −1.5 with high probability, as shown in Figure 4. This result confirms the statistics discussed in Subsection IV.A. In the CDF 9/7-tap wavelet filter, α is an irrational number which is −1.586134342... (Cohen et al., 1992; Daubechies and Sweldens, 1998). In our proposed methods, with each image, we can find the optimal α which is simply a rational number. For example, α_{opt} of LYNDA and GRANDMA images are −2.6 and −1.68, respectively. Moreover, we obtain improvement PSNR with the optimal α for each image. For instance, using filter optimization in our previous work, we obtained up to 0.11 dB of PSNR gain for FAXBALLS image that is one of JPEG2000 standard images (Quan and Ho, 2009). Therefore, our methods result in a doubled efficiency.

Table II shows the maximum PSNR gain. The maximum PSNR gains are 1.36 dB and 1.48 dB on average using the median lifting scheme and using the combination of filter optimization with median lifting, respectively. We also obtain the PSNR improvements as 1.37 dB and 1.42 dB on average for BIRD image. In addition, we get up to 2.05 dB on maximum PSNR gain at 0.125 bpp for GRANDMA image.

We consider the maximum and average PSNR gains achieved by our proposed methods and by methods of Ding et al. and Liu and Ngan. However, our proposed methods provide many advantages for edge-dominant images, while their methods provide those for images with rich orientation features such as BARBARA and WOMEN images (Ding et al., 2007). It is unfair if we make a comparison for experimental results of different databases. Hence, we show the results of our methods and those of the two existing works.

Compared to the CDF 9/7-tap wavelet filter of JPEG2000, the maximum PSNR gain of Ding et al. (2007) method and that of Liu and Ngan (2008) method are 1.51 dB and 1.79 dB, respectively. Our proposed methods achieve 1.66 dB gain for the median lifting

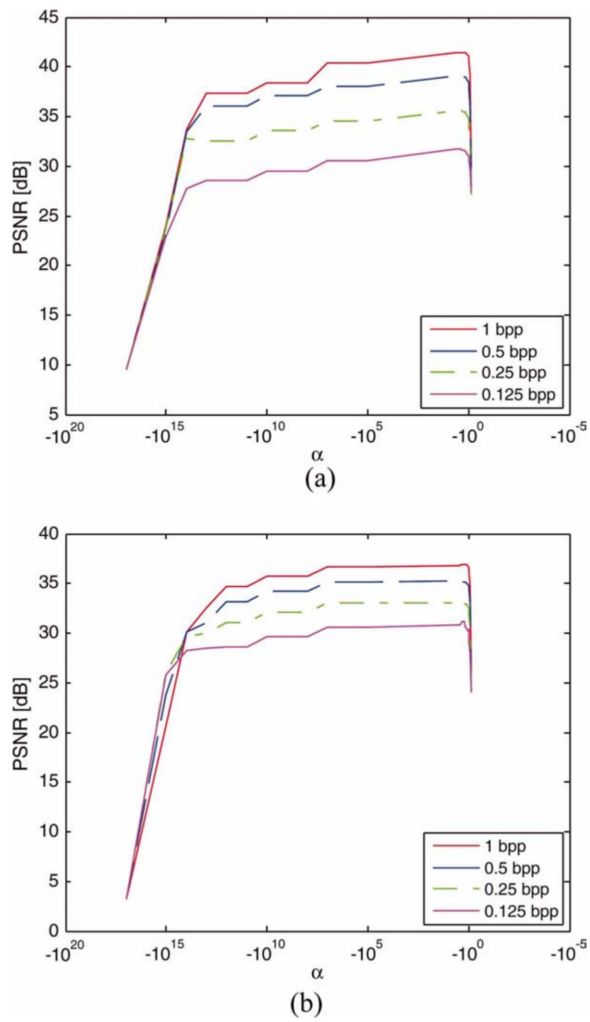

Figure 5. PSNR survey with different rates for (a) LYNDA image and (b) GRANDMA image. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table II. Comparison of PSNR values

Image	Rate (bpp)	Ref. I[a] (dB)	Ref. II (dB)	Method A (dB)	Method B (dB)	$\delta PSNR_{AI}$ (dB)	$\delta PSNR_{BI}$ (dB)	$\delta PSNR_{AII}$ (dB)	$\delta PSNR_{BII}$ (dB)
BIRD	1	36.27	36.27	37.76	37.77	+1.49	+1.5	+1.49	+1.5
($\alpha_{opt} = -1.65$)	0.5	32.6	32.6	34.17	34.19	+1.57	+1.59	+1.57	+1.59
	0.25	29.49	29.46	30.79	30.87	+1.3	+1.38	+1.33	+1.41
	0.125	26.56	26.53	27.69	27.71	+1.13	+1.15	+1.16	+1.18
CLOWN	1	32.5	32.48	33.73	33.72	+1.23	+1.22	+1.25	+1.24
($\alpha_{opt} = -1.55$)	0.5	29.58	29.55	30.42	30.47	+0.84	+0.89	+0.87	+0.92
	0.25	27.56	27.55	27.63	27.63	+0.07	+0.07	+0.08	+0.08
	0.125	25.56	25.54	25.53	25.53	−0.03	−0.03	−0.01	−0.01
GRANDMA	1	36.35	36.32	36.89	36.89	+0.54	+0.54	+0.57	+0.57
($\alpha_{opt} = -1.68$)	0.5	34.45	34.42	35.13	35.15	+0.68	+0.7	+0.71	+0.73
	0.25	32.2	32.19	32.96	32.98	+0.76	+0.78	+0.77	+0.79
	0.125	29.12	29.11	30.58	31.16	+1.46	+2.04	+1.47	+2.05
LYNDA	1	40.38	40.36	41.42	41.48	+1.04	+1.1	+1.06	+1.12
($\alpha_{opt} = -2.6$)	0.5	37.38	37.35	38.98	39.06	+1.6	+1.68	+1.63	+1.71
	0.25	33.8	33.79	35.46	35.56	+1.66	+1.76	+1.67	+1.77
	0.125	30.42	30.4	31.54	31.71	+1.12	+1.29	+1.14	+1.31
EINSTEIN	1	30.6	30.6	31.17	31.18	+0.57	+0.58	+0.57	+0.58
($\alpha_{opt} = -3.2$)	0.5	28.7	28.7	28.63	28.85	−0.07	+0.15	−0.07	+0.15
	0.25	25.49	25.46	25.85	26.23	+0.36	+0.74	+0.39	+0.77
	0.125	23.77	23.74	23.64	23.73	−0.13	−0.04	−0.1	−0.01
ELAINE	1	32.47	32.43	32.78	32.78	+0.31	+0.31	+0.35	+0.35
($\alpha_{opt} = -1.59$)	0.5	31.44	31.41	31.66	31.66	+0.22	+0.22	+0.25	+0.25
	0.25	29.87	29.84	30.14	30.14	+0.27	+0.27	+0.3	+0.3
	0.125	28.62	28.61	28.32	28.32	−0.3	−0.3	−0.29	−0.29
MRI	1	31.46	31.45	32.35	32.37	+0.89	+0.91	+0.9	+0.92
($\alpha_{opt} = -1.52$)	0.5	28.91	28.9	29	29.02	+0.09	+0.11	+0.1	+0.12
	0.25	26.61	26.59	26.74	26.76	+0.13	+0.15	+0.15	+0.17
	0.125	24.01	23.98	24.08	24.19	+0.07	+0.18	+0.1	+0.21
AVERAGE						+0.67	+0.75	+0.69	+0.77

[a] Ref. I, CDF 9/7 of JPEG2000; Ref. II, Guangjun's method; Method A, proposed method using median lifting scheme; Method B, proposed method using combination of filter optimization and median lifting.

scheme and 2.04 dB gain for the combination of filter optimization with median lifting, as shown in Table II.

Compared to the CDF 9/7-tap wavelet filter of JPEG2000, the average PSNR gain obtained by Ding et al. (2007) method is 0.57 dB, while that obtained by Liu and Ngan (2008) method is 0.69 dB. In our methods, as shown in Table II, the gain of the median lifting scheme is 0.67 dB and that of the combination of filter optimization with median lifting is 0.75 dB.

Figure 6 displays the rate distortion (RD) curves that clearly indicate the significant PSNR improvement of our methods over the two reference methods. As discussed in Section I, Guangjun's method simplified the filter coefficients by using the fixed value

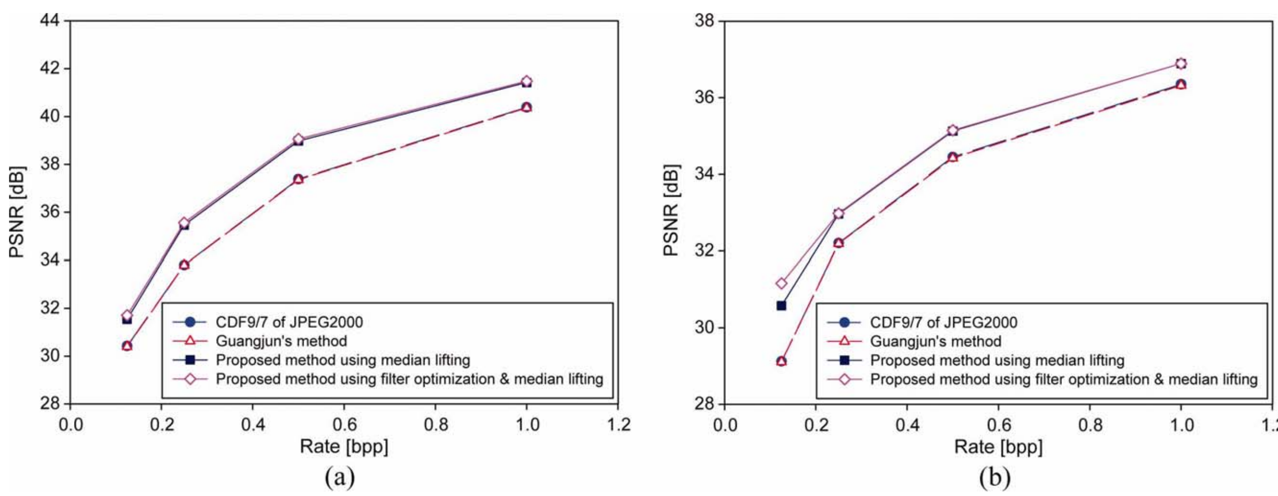

Figure 6. RD curves for (a) LYNDA image and (b) GRANDMA image. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

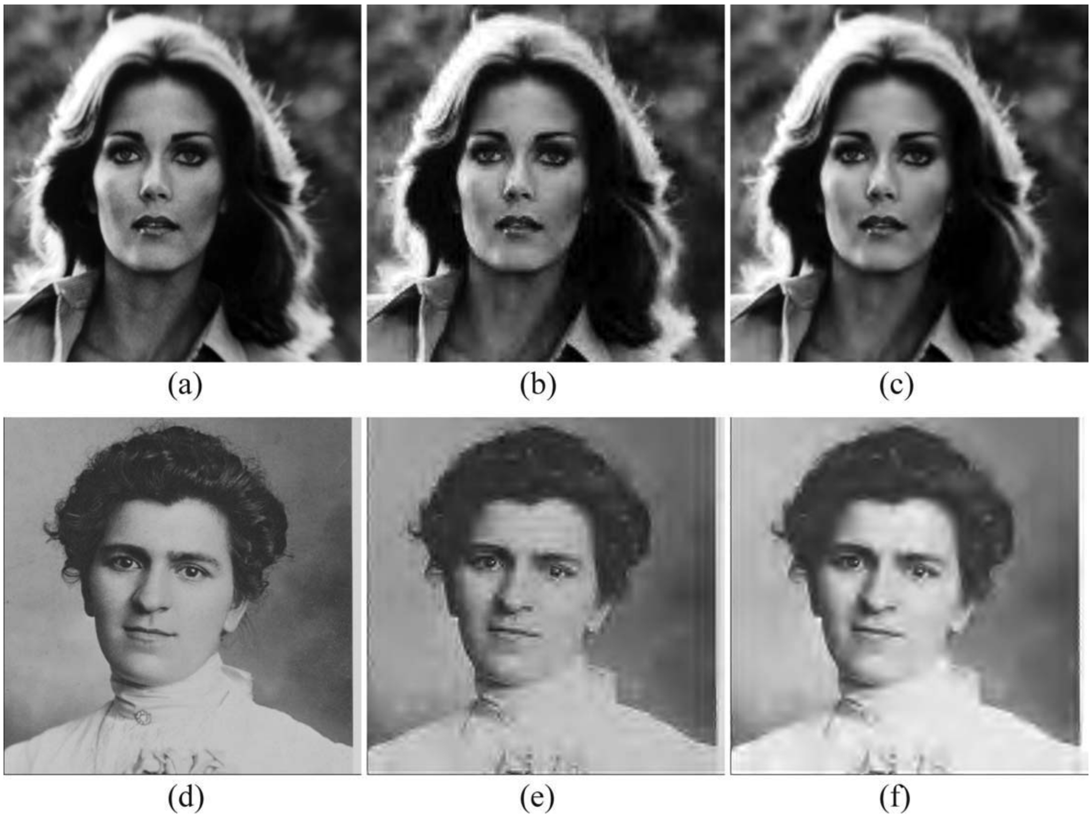

Figure 7. Visual quality comparison for LYNDA image at 0.25 bpp (top row) and GRANDMA image at 0.125 bpp (bottom row). (a, d) Original images. (b) Conventional lifting scheme for LYNDA image (33.8 dB). (c) Proposed lifting scheme for LYNDA image (35.56 dB). (e) Conventional lifting scheme for GRANDMA image (29.12 dB). (f) Proposed lifting scheme for GRANDMA image (31.16 dB).

$\alpha = -1.5$. However, the performance of Guangjun's method is similar to that of the CDF-9/7 method of JPEG2000 (Guangjun et al., 2001). As shown in Figure 6, the proposed methods outperform two reference methods: the CDF-9/7 method of JPEG2000 and Guangjun's method.

Compared to RD curves in Figure 6, the combination of filter optimization and median lifting performs slightly better than the median lifting scheme does. However, both of our proposed methods outperform the two reference methods.

D. Comparison of Visual Quality and Complexity. We examine the output image qualities of different methods. Figure 7 compares their visual quality. As PSNR is not the ultimate judge in image quality (Liang et al., 2003), we can only decide on the best method after proving the improvement of PSNR and visual quality. We compare the visual quality of decoded images which result from the proposed lifting scheme and the conventional lifting scheme in the CDF-9/7 of JPEG2000. Figure 7 compares visual quality in detail. This figure indicates the output image qualities from using different methods. The first column shows the original images. Figure 7 shows that compared with the conventional lifting scheme, our proposed lifting scheme resulted in significant visual quality improvement.

As the median lifting scheme and the proposed lifting scheme use the median operator, which provides many advantages for edge-dominant images (Jansen and Oonincx, 2005), the proposed lifting scheme gives much better ability of preserving edges compared with the conventional lifting scheme. Indeed, we first notice that the quality of the third column images is much better than that of the second column images. In fact, the ringing artifacts can be seen in the face, chin, and boundaries of objects in the images. Experiment results show that the ringing artifacts are less severe in the output images of the proposed lifting scheme than in those of the conventional lifting scheme. Therefore, our proposed lifting scheme outperforms the conventional lifting scheme in terms of visual quality.

As our lifting scheme is the combination of filter optimization and median lifting, it has higher computational complexity than the CDF-9/7 method of JPEG2000 and Guangjun's method. However, our lifting scheme reduces the complexity of wavelet filter coefficients since their irrational coefficients were replaced by rational coefficients, compared to the CDF-9/7 method of JPEG2000, and it has similar complexity of filter coefficients as Guangjun's method.

V. CONCLUSIONS

In this article, we proposed an efficient lifting scheme as a combination of filter optimization and median lifting for edge-dominant images. Because we replaced irrational coefficients of wavelet filters by rational coefficients, our proposed methods reduced the complexity of wavelet filter coefficients. In addition, we determined PSNR curve as a concave function of α. Experiment results demonstrated that compared to the well-known CDF-9/7 method of JPEG2000, the proposed methods for edge-dominant images significantly improved the PSNR. Finally, the proposed lifting scheme improved the visual quality.

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